

## FUZZY MODEL FOR ENERGY LOSSES CALCULATION IN LOW VOLTAGE DISTRIBUTION NETWORKS

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**Abstract:** The proper electric energy losses computing is one of the most complex problems in power distribution system analysis, demanding consideration of many factors. The accessibility and credibility of data used in calculations is of the greatest importance here. The careful analysis of system losses is desirable in this respect. This article presents the mathematical model of electric energy losses in low voltage networks with application of fuzzy sets theory. Using a fuzzy approach makes possible to improve loss calculations accuracy. Theoretical statement is illustrated by an example which corresponds to Polish distribution system.

**Keywords:** Energy losses, Fuzzy sets, Power distribution systems

### 1. INTRODUCTION

The occurrence of electric energy losses is connected with the energy production, transmission and distribution processes. Specific physical phenomena taking place during electric energy transmission and conversion in the energy system have an influence on its losses.

Electric energy losses occurring in the electric power networks are very essential element of the every power utility balance. In Polish conditions they reach dozen or so per cent, in extreme cases they may gain a few dozen per cent the whole energy flowing through all the network levels [2,4,6]. The biggest participation in arising of electric energy losses belongs to the medium and low voltage distribution networks, which determine about 80 per cent all of the losses coming into being [5].

The limiting of the costs of energy losses occurring in distribution power networks could be potentially the source of significant savings. The fundamental problem is correct estimating different kinds of energy losses taking place in distribution networks. The results and the precision of electric energy loss calculation depend mainly on types and the quality of available input data.

### 2. TECHNICAL ENERGY LOSSES IN LOW VOLTAGE DISTRIBUTION NETWORKS

Technical losses are physical electric energy losses arising in electric power network elements and connected with electric energy transmission and distribution processes.

Technical losses in the medium and low voltage distribution networks consist of: load losses in overhead and cable medium voltage lines, distribution transformer windings, overhead and cable low voltage lines, low voltage services, internal feeder lines and idling losses in

voltage coils of the meters, medium voltage lines and distribution transformer cores.

Energy losses in municipal distribution power networks are being considered in this paper. In Poland they deliver energy to living, commerce, service and industry consumers situated in the urban areas. They consist of low and medium voltage networks. The low voltage networks are fed from 15/0.4 kV transformers. They are mostly designed as a cable ones. There are low voltage overhead lines in peripheral quarters and detached houses estates. The distribution network form and way of the operation is determined by a type of consumers, a load density and an urban structure. In case of large agglomerations with dense building cable lines are conducted along the streets grid. Final or arterial cable joints connect consumers and the network. In case of important receivers, par example high buildings with lifts, boiler houses, department stores, offices or out-patient clinics, a ring-shaped supply system with use of automatic loss-of-voltage tripping is applied. Overhead and low voltage cable lines feeding streets lighting circuits are also the element of the urban distribution network. There are control devices, just like clocks or photocells by the side of low voltage in 15/0.4 kV transformer substations. They switch on or switch off all the lamps placed along the streets.

The elementary segment of the low voltage distribution network considered in this paper is the fragment consisting of one 15/0.4 kV transformer substation, overhead or cable feeder lines coming out from it, internal feeder lines, electric energy meters and energy capacitors installed by the side of low voltage.

The total technical losses of electric energy, for one distribution network section including one 15/0.4 kV distribution transformer with low voltage lines and consumers, consist of [5,6]:

$$\Delta A_t = \Delta A_{Fe} + \Delta A_{Cu} + \Delta A_u + \Delta A_L + \Delta A_{wLz} + \Delta A_{Lc} + \Delta A_k \quad (1)$$

where:

- $\Delta A_t$  - technical electric energy losses in low voltage distribution network,
- $\Delta A_{Fe}$  - electric energy losses in transformer iron core,
- $\Delta A_{Cu}$  - load energy losses in transformer,
- $\Delta A_u$  - electric energy no-load losses in low voltage lines,
- $\Delta A_L$  - electric energy load losses in low voltage lines,
- $\Delta A_{wLz}$  - electric energy losses in internal feeder lines,
- $\Delta A_{Lc}$  - electric energy losses in meters,
- $\Delta A_k$  - dielectric losses of energy in capacitors of the low voltage network.

### 3. THE SIMPLIFIED LOW VOLTAGE NETWORK MODEL FOR ELECTRIC ENERGY LOSSES CALCULATIONS

The exemplary segment of the urban distribution low voltage network is being considered. The system is fed by one transformer substation working with one 15/0,4 kV transformer. Low voltage feeder lines deliver electric energy to individual receiving buses. In case of buildings meant for many families cable joints are used. If we deal with detached houses, individual services or cable joints of separated consumers groups should be applied. Each reception centre is characterised by load current and feeder line's resistance between the centre and the transformer substation.

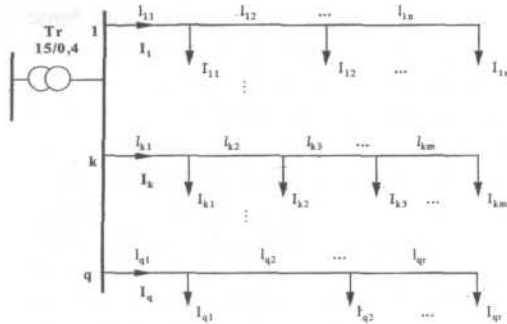


Fig. 1. The exemplary segment of the electric energy distribution low voltage network fed by one transformer substation:  $l_{11}, \dots, l_{qr}$  — lengths of individual feeder lines sections;  $I_{11}, \dots, I_{kr}$  - load currents in individual receiving buses;  $I_1, \dots, I_q$  - load currents coming into individual feeder lines;  $k$  - the number of feeder lines coming out of the transformer substation;  $n, m, r$  - the number of receiving buses in individual feeder lines

All of the components of technical losses, for considered system, can be calculated as shown below:

$$\Delta A_{Fe} = \frac{\Delta P_{FeN} T_m U_S^2}{U_N^2} \quad (2)$$

where:

- $\Delta P_{FeN}$  - nominal no-load losses in the transformer core,
- $T_m$  - time of the transformer load duration,
- $U_S$  - average voltage on the transformer buses,
- $U_N$  — nominal network voltage.

$$\Delta A_{Cu} = \Delta P_{CuN} B^2 \tau_m k_T \quad (3)$$

where:

- $\Delta P_{CuN}$  - nominal load losses of transformer,
- $B$  — transformer load rate,
- $\tau_m$  - time of maximal losses duration,
- $k_T$  - temperature coefficient.

(4)

$$\Delta A_u = c_u L T_m$$

where:

- $L$  - the total length of low voltage lines fed by considered transformer substation,
- $c_u$  - coefficient of unit line losses.

$$\Delta A_L = 3 \tau_m \sum_{i=1}^n I_{\max i}^2 R_i \quad (5)$$

where:

- $I_{\max i}$  - maximal current flowing in  $i$  - th line,
- $R_i$  - resistance of  $i$  - th feeder line,
- $n$  - number of feeder lines,
- $\tau_m$  - time of maximal load losses duration.

$$\Delta A_{wLz} = c_2 A_k \quad (6)$$

where:

- $A_k$  - electric energy flowing through considered termination assembly,
- $c_2$  - coefficient of the relation between losses in services and  $A_k$  energy.

$$\Delta A_{1c} = (c_1 n_1 + c_3 n_3) T_m \quad (7)$$

- $c_1$  - unit electric energy losses in 1 - phase meters,
- $n_1$  - the number of 1 - phase meters installed,
- $c_3$  - unit electric energy losses in 3 - phase meters,
- $n_3$  - the number of 3 - phase meters installed.

The simplification of the model consists in transforming the circuit from Fig.1 into the equivalent circuit shown in Fig. 2. The transformation condition is the equity of total power losses in the real feed system and the equivalent one [1]. Apart from that the equivalent load current  $I_{eq}$  is being considered to be equal transformer load current  $I_{Tr}$

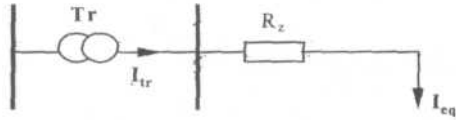


Fig. 2. The equivalent chart of the circuit shown in Fig.1:  $R_z$  - the equivalent resistance of -all feeder lines coming out of the transformer substation;  $I_{eq}$  - the equivalent load current;  $I_{Tr}$  - the transformer load current.

Therefore following equations can be written:

$$I_{eq} = I_1 + I_2 + \dots + I_q = \sum_{i=1}^q I_i \quad (8)$$

$$R_z = \frac{\sum_{i=1}^k I_i^2 R_{iz}}{I_{eq}^2} \quad (9)$$

where:

$R_z$  - equivalent resistance of all feeder lines, by which power losses in equivalent circuits are equal,

$I_{eq}$  - equivalent load current.

Having the information about the value of the electric energy consumed by individual consumers connected to receiving buses (par example on the basis of meters registration) and having the knowledge about the amount of electric energy flowing through the transformer substation, it is possible to calculate values of load currents in individual feeder lines, depending on transformer load current, as follows:

$$I_{eqkmax} = \alpha_k I_{Tr} \quad (10)$$

where:

$I_{eqkmax}$  ~ the equivalent load current flowing through k-th feeder line,

$I_{Tr}$  - transformer load current,

$\alpha_k$  - the ratio of k - th maximal equivalent current to transformer load current, while:

$$\alpha_k = \frac{I_{eqkmax}}{I_{Tr}} = \frac{a_k \bar{A}}{A_0} = \frac{a_k}{a} \quad (11)$$

where:

$a_k$  - number of consumers fed by k - th feeder line,

$a$  - number of all consumers fed by given distribution substation,

$\bar{A}$  - average consumption of electric energy for consumers fed by given transformer substation,

$A_0$  - electric energy flowing through the distribution substation.

Because of significant shortage of data and measurements in municipal distribution electric energy low voltage networks the electric energy losses estimation process in those networks is led in conditions of limited access to information. The limit concerns individual feeder lines and receiving buses load knowledge and the value of electric energy consumed by specified consumers groups, as well as the knowledge of technical parameters of distribution low voltage network elements.

The considerations above lead to the conclusion that all the quantities appearing in (I)-(II) are uncertain numbers, being characterised by fuzziness. On the basis of owned information about network parameters, data about consumers and the value of electric energy consumed by individuals consumers groups the numbers fuzziness ranges can be estimated.

One of the ways of fuzzy quantity descriptions is to show them in the form of triangle type numbers. In this case to every quantity, one real value may be assigned. As a rule, the assigned value is not known or is known approximately. However, it is possible to specify the number range, in which the number may be situated with enough certainty for practical aims. During the ranges limits estimating it is possible to use both available information about average values of quantities considered, and catalogue data taken from producers. The important source to the point may be also the experience and the practice of the technical personnel employed in district dispatch office of individual distribution supply company.

With such assumption all quantities from equations (1 ÷ 11) may be written in the form as follows:

$$W = (w_1, w_2, w_3) \quad (12)$$

where:  $w_1, w_2, w_3$  — parameters of fuzzy number  $W$ .

For this reason individual components of the mathematical model of the electric energy losses in municipal power distribution low voltage networks may be shown in the form of triangle type fuzzy numbers with parameters put together in Table 1.

Total technical electric energy losses in fuzzy form are represented as a sum of individual components of those losses shown as fuzzy ones. Hence the equation to represent the fuzzy model of electrical energy losses in low voltage municipal power distribution networks is:

$$\tilde{\Delta A}_t = \tilde{\Delta A}_{Fe} + \tilde{\Delta A}_{Cu} + \tilde{\Delta A}_u + \tilde{\Delta A}_L + \tilde{\Delta A}_{wiz} + \tilde{\Delta A}_{LC} \quad (13)$$

In the equation (13) individual symbols signify the same quantities as in formula (1), but shown as fuzzy ones.

TABLE 1.

Parameters of electric energy losses in municipal distribution low voltage networks shown in the form of triangle type fuzzy numbers

Quantity	Symbol	The triangle form of fuzzy number
Nominal no-load losses in the transformer iron core	$\Delta P_{FeN}$	$(\Delta P_{FeN} - \delta\Delta P_{FeN}, \Delta P_{FeN}, \Delta P_{FeN} + \delta\Delta P_{FeN})$
Nominal load transformer losses	$\Delta P_{CuN}$	$(\Delta P_{CuN} - \delta\Delta P_{CuN}, \Delta P_{CuN}, \Delta P_{CuN} + \delta\Delta P_{CuN})$
Time of load duration	$T_m$	$(T_m - \Delta T_m, T_m, T_m + \Delta T_m)$
Relative time of maximal load duration	$t_{sm}$	$(t_{sm} - \Delta t_{sm}, t_{sm}, t_{sm} + \Delta t_{sm})$
Voltage during maximal load	$U_s$	$(U_s - \Delta U_s, U_s, U_s + \Delta U_s)$
Average voltage in the substation	$U_{sm}$	$(U_{sm} - \Delta U_{sm}, U_{sm}, U_{sm} + \Delta U_{sm})$
Power coefficient during maximal load	$\cos\varphi_s$	$(\cos\varphi_{smin}, \cos\varphi_s, \cos\varphi_{smax})$
Average coefficient of power	$\cos\varphi_{sm}$	$(\cos\varphi_{sm,min}, \cos\varphi_{sm}, \cos\varphi_{sm,max})$
Coefficient of transformer windings resistance conversion because of temperature	$k_T$	$(k_{Tmin}, k_T, k_{Tmax})$
Transformer load rate	$B$	$(B - \Delta B, B, B + \Delta B)$
Time of maximal losses-duration	$\tau_m$	$(\tau_m - \Delta\tau_m, \tau_m, \tau_m + \Delta\tau_m)$
K – th feeder line load current	$I_{eqk}$	$(I_{eqkmin}, I_{eqk}, I_{eqkmax})$
Length of i – th section of k – th feeder line	$l_{ik}$	$(l_{ik} - \Delta l, l_{ik}, l_{ik} + \Delta l)$
Energy consumed by k – th line consumers	$A_k$	$(A_{kmin}, A_k, A_{kmax})$
Coefficient of unit line losses	$c_u$	$(c_{umin}, c_u, c_{umax})$
Coefficient describing energy losses portion in services	$c_2$	$(c_{2min}, c_2, c_{2max})$
Energy unit losses in 1 – phase meters	$c_1$	$(c_{1min}, c_1, c_{1max})$
Energy unit losses in 3 – phase meters	$c_3$	$(c_{3min}, c_3, c_{3max})$

After summation of all equation (13) components one receives the following fuzzy form of mathematical model of technical electric energy losses in low voltage municipal distribution networks (14):

$$\begin{aligned}
\Delta \tilde{A}_t = & \{ \{ \{ \frac{1}{U_N^2} (\Delta P_{FeN} - \delta\Delta P_{FeN})(T_m - \Delta T_m)(U_{sm} - \Delta U_{sm})^2 \} + [(\Delta P_{CuN} - \delta\Delta P_{CuN})(\tau_m - \Delta\tau_m)k_{Tmin}(B - \Delta B)^2] + \\
& + [(T_m - \Delta T_m)c_{umin}(L - \Delta L)] + 3 \sum_{i=1}^n [(I_{imin})^2(R_i - \Delta R)(\tau_m - \Delta\tau_m)] + [c_{2min}A_{kmin}] + [(n_1c_{1min} + n_3c_{3min})(T_m - \Delta T_m)] \}, \\
& \{ \{ \Delta P_{FeN}T_mU_{sm}^2 \} + [ \Delta P_{CuN}\tau_m k_T B^2 ] + [c_u L T_m] + [(I_i)^2 R_i \tau_m] + [c_2 A_k] + [(n_1 c_1 + n_3 c_3) T_m] \}, \\
& \{ \{ \frac{1}{U_N^2} (\Delta P_{FeN} + \delta\Delta P_{FeN})(T_m + \Delta T_m)(U_{sm} + \Delta U_{sm})^2 \} + [(\Delta P_{CuN} + \delta\Delta P_{CuN})(\tau_m + \Delta\tau_m)k_{Tmax}(B + \Delta B)^2] + \\
& + [(T_m + \Delta T_m)c_{umax}(L + \Delta L)] + 3 \sum_{i=1}^n [(I_{imax})^2(R_i + \Delta R)(\tau_m + \Delta\tau_m)] + [c_{2max}A_{kmax}] + [(n_1 c_{1max} + n_3 c_{3max})(T_m + \Delta T_m)] \} \} \}
\end{aligned} \quad (14)$$

## 5. THE COMPUTATIONAL EXAMPLE

The exemplary fragment of urban distribution low voltage network fed by one distribution transformer of 15/0.4 kV and 400 kVA rating (Fig. 3). Consumers (blocks of flats) by the low voltage side are fed by 4 cable lines YAKY 4x120 operating in a radial system of 605 m total length [6]. Parameters values used in calculations are put together in Table 2.

Electric energy losses computed on the basis of

presented fuzzy mathematical model are represented by fuzzy triangle type number (1.043 kWh; 1.247 kWh) with the central value equal to 1.137 kWh. Fig. 4 presents the graphic image of the fuzziness of the electric energy losses result value.

The final result of the calculations estimates the range, in which the value of electric energy losses is supposed to be situated. The width of the range depends on the fuzziness extent of individual parameters forming energy losses mathematical model.

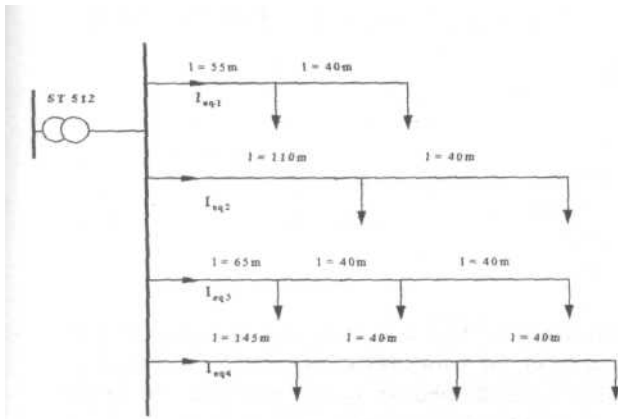


Fig. 3. The exemplary segment of urban distribution low voltage network

TABLE 2  
Assumed parameter values of the fuzzy model of energy losses

Quantity	Range of fuzziness	Quantity	Range of fuzziness
$\Delta P_{FeN}$	[0.986 ; 1.038 ; 1.090]	$I_{Tr}$	[0.772 ; 0.902 ; 1.033]
	[- 5 % ; + 5 %]		[- 14 % ; + 14 %]
$\Delta P_{CuN}$	[5.108 ; 5.377 ; 5.646]	$A_k$	[13728 ; 14451 ; 15174]
	[- 5 % ; + 5 %]		[- 5 % ; + 5 %]
$T_m$	[675.1 ; 696.0 ; 716.9]	$C_u$	[1.4 ; 1.7 ; 2.2]
	[- 3 % ; + 3 %]		[- 18 % ; + 29 %]
$U_{sm}$	[0.352 ; 0.371 ; 0.389]	$c_2$	[0.0027 ; 0.003 ; 0.0032]
	[- 5 % ; + 5 %]		[- 10 % ; + 7 %]
$\cos\phi_s$	[0.895 ; 0.942 ; 0.989]	$c_1$	[1.4 ; 1.5 ; 1.7]
	[- 5 % ; + 5 %]		[- 7 % ; + 13 %]
$k_T$	[1.045 ; 1.100 ; 1.155]	$c_3$	[2.5 ; 3.4 ; 4.1]
	[- 5 % ; + 5 %]		[- 27 % ; + 21 %]

## 6. CONCLUDING REMARKS

Considerations carried out in the paper confirm the possibility of creating the mathematical model based on the fuzzy sets approach, for electric energy losses estimating in distribution electric energy low voltage networks. The application of fuzzy analysis rules made possible to receive satisfactory estimating results, in spite of the significant measurements shortage occurring in distribution feed systems.

The way of estimation of technical electric energy losses in distribution low voltage networks permits an effective use of available information concerning loads and technical parameters of individual feeder lines and receiving buses and the value of the electric energy consumed.

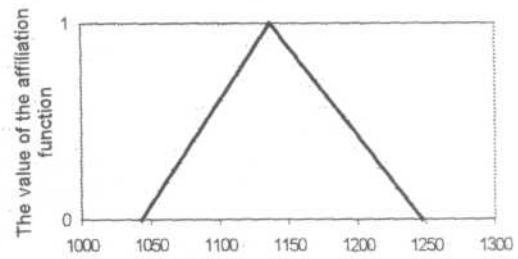


Fig. 4. The graphic image of electric energy losses fuzziness in the considered system.

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## BIOGRAPHIES

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