THE AMBIGUITY OF THE RESULTS OF THE STRICT ADJUSTMENT OF HORIZONTAL GEODETIC NETWORKS

MSc. Krzysztof Butryn

Ph.D Elżbieta Jasińska

Assoc. Prof. Ph.D. Edward Preweda

AGH University of Science and Technology, Krakow, **Poland**

ABSTRACT

The algorithm of the strict adjustment of horizontal geodetic networks using least square adjustment method is well known and widely described in the literature. Referring to the Regulation of the minister of administration and digitization of 14 February 2012 on geodetic, gravimetric and magnetic control networks, which is in force in Poland, the authors draw attention to the important provision contained in this Ordinance: Observations should be adjusted using a rigorous adjustment, based on least square adjustment method, assuming zero variance for references points.

Assumption of zero standard deviations for references points causes that, despite network adjustment process is rigorous, results obtained, both adjusted coordinates and the assessment of the accuracy of the coordinates, depend on the method of establishing the network, the number of points and the actual accuracy of references points. On the example of the small test network, there was demonstrated how important differences can be.

The only solution to the problem of the ambiguity of the results of strict network adjustment is taking into account the accuracy of references points. It should be noted that the coordinates of references points can be a result of adjustment of various networks in various time periods, so they can be determined with various precision. Taking into account this fact can only be done by determining reliable relationship between weights of observations and coordinates of references points.

Keywords: geodetic network, least squares method, inerrancy establish points, coordinates

INTRODUCTION

The concept of strict adjustment of geodetic networks is very well known in the surveyors' community. It is identified, however, only using one of the strict methods of solution of the functional stochastic model. In surveying issues there is usually used the least square adjustment method. It should be noted that using a references to the different points which coordinates are known, despite the strict mathematical algorithm, different values of estimators are obtained [4]. This applies to both the coordinates and parameters describing the accuracy of the designation. [11], [12], [13]. In addition, the coordinates of references points are often determined with less accuracy, than the accuracy achievable during the current measurements.

The assumption of zero standard deviations for references points, which in Polish conditions is defined by Regulation [14], result in the imposition of observation “amendments" resulting from this assumption that is not correct in terms of content. These "corrections" in fact worsen the results of observations, which is reflected in the results of geodetic studies. This results in various types of problems with the correct fixing of the geometry of different objects. Issues related to this problem are described, among others, in this works [2], [6], [7], [8]. On the issue of network adjustment there is also connected closely the issue of the determination of displacements and deformations of the land surface, and various engineering objects. These issues are discussed for example in [9], [10] and [11]. Due to the requirement to obtain a high precision, resulting primarily from the need to ensure the safety of the building objects, taking into account the accuracy of references points should never be ignored.

Specific methods of geodetic observations adjustment, describing among other things, the opportunity to take the time and use pseudoinverse of matrix, are described in [3], [13]. The assumption of zero variance for references points also does not include a correlation between the accuracy of the coordinates of references points, which significantly affect the results of the evaluation accuracy.

In this paper, the authors present the most important results of the calculations and the most important observations and conclusions obtained for the horizontal network consisting of five points. Detailed and wider discussion of the problem is shown in [5].

The purpose of this paper is to determine the impact on the determined coordinates of network points and on the assessment of their accuracy, has the method of geodetic connection of network and taking into account inaccuracies of references points.

MATERIALS AND CALCULATIONS

Contemplated horizontal network consists 5 points. It is assumed that the purpose of discussion is to determine the coordinates of points 1 and 2, assuming that the references can be based on three points, numbered 10, 20 and 30. Coordinates of references points and observations along with their standard deviations are in tables 1 and 2. Calculations were conducted according to the algorithms described in [13].

Table 1. Coordinates of references points

Point number	X [m]	Y [m]
10	2247,761	5661,420
20	4717,011	4942,448
30	3517,458	7091,148

Assuming that different references points have zero standard deviations (respectively - three or two points, one point, and the azimuth of selected side), using parametric method there were set estimators of points coordinates and parameters describing their accuracy. The selected values are presented in Table 3.

Table 2. Measured lengths and angles

Sign.	From - To	Length [m]	Mean error [m]
d_1	1 – 10	759,233	0,010
d_2	1 – 2	1428,231	0,010
d_3	1 – 20	2139,175	0,010
d_4	1 – 30	1204,590	0,010

Sign.	L	C	R	α [g]	σ [cc]
α_1	10	1	20	129,7200	15
α_2	10	2	20	38,7642	20
α_3	2	1	10	76,5082	20
α_4	2	30	20	126,3520	20
α_5	20	10	2	141,7622	20
α_6	30	20	1	32,5620	15

Table 3. Parameters of error ellipses and error circles – error-free connection assumption

Point signature	Parameters of the constant probability density ellipse P=0,39. (A, B – axes, φ - azimuth of A axis)			Radius of error circle	Radius of error hypersphere
	A [mm]	B [mm]	φ [g]	r [mm]	R [mm]
Variant 1) zero standard deviation for coordinates of points 10, 20 and 30					
1	13,1	11,2	143,03	12,1	18,2
2	47,7	20,1	57,48	30,9	
Variant 2) zero standard deviation for coordinates of points 10 and 20					
1	19,7	15,1	83,08	17,2	32,8
2	58,4	24,4	58,42	37,7	
30	209,0	26,4	165,88	74,2	
Variant 3) zero standard deviation for coordinates of points 10 and 30					
1	14,9	4,4	149,43	8,1	10,3
2	33,9	10,7	36,14	19,1	
20	50,3	9,9	80,96	22,3	
Variant 4) zero standard deviation for coordinates of points 20 and 30					
1	7,1	6,8	185,84	7,0	9,0
2	25,0	9,8	58,96	17,7	
10	13,7	9,5	123,85	11,4	
Variant 5) zero standard deviation for coordinates of points X_{10}, Y_{10}, X_{20}					
1	14,8	5,2	141,37	8,8	11,1
2	24,7	11,0	27,7	16,5	
20	31,6	0,0	100,00	-	
30	64,5	8,7	164,04	23,7	
Variant 6) zero standard deviation for coordinates of points X_{10}, Y_{10}, Y_{20}					
1	6,5	5,3	139,41	5,9	9,3
2	16,6	8,2	57,98	11,7	
20	9,2	0,0	0,00	---	
30	59,5	7,5	165,87	21,1	
Variant 7) zero standard deviation for coordinates of points X_{20}, Y_{20}, Y_{10}					
1	7,9	5,0	139,55	6,3	9,31
2	17,4	7,5	54,44	11,4	
10	9,2	0,0	0,00	-	
30	59,9	8,9	164,48	23,1	

Similar calculations were conducted assuming fallacy of the references points (points number 10, 20 and 30). For references purposes, standard deviations of coordinates were taken at the level of 1, 5 and 10 centimeters. Selected parameters of assessing the accuracy are summarized in Table 4.

Table 4. Parameters of error ellipses and error circles – inaccuracy of references points included

Point signature	Parameters of the constant probability density ellipse P=0,39. (A, B – axes, φ - azimuth of A axis)			Radius of standard deviation circle	Radius of standard deviation hypersphere
	A [mm]	B [mm]	φ [g]	r [mm]	R [mm]
Variant 8) Coordinates of references points ±1 cm					
1	13,8	12,3	157,43	13,1	12,5
2	40,8	18,2	55,76	27,3	
10	11,7	10,9	119,04	11,3	
20	12,7	11,2	81,52	11,9	
30	12,7	10,8	166,12	11,7	
Variant 9) Coordinates of references points ±5 cm					
1	21,4	19,7	185,76	20,5	11,8
2	39,7	21,5	52,77	29,2	
10	24,2	20,0	110,76	22,0	
20	28,6	20,9	75,67	24,4	
30	27,7	19,9	171,12	23,5	
Variant 10) Coordinates of references points ±10 cm					
1	30,9	30,6	12,81	30,8	12,1
2	55,0	31,0	57,70	41,3	
10	38,1	29,7	111,23	33,6	
20	43,7	30,7	67,0	36,7	
30	40,4	30,1	180,91	34,9	

In addition to the obtained various parameters for assessing the accuracy, resulting of the way of connection and the assumption of correctness or no-correctness of references points coordinates, there are different values of estimators of the determined points coordinates. The differences in the results will be the greater, the greater is the discrepancy between the actual accuracy of the designation of references points compared to the precision of observations. Given that the geodetic connection is often based on the points determined by a much lower grade of measuring equipment in comparison with accuracies that are possible to achieve today, ignoring this fact leads to a situation in which the finally crafted observations receive "fix", which is often several times greater than actual measurement accuracy. Such a situation should not take place in precision surveying and may lead to serious consequences, including formal and legal.

Table 5 summarizes differences between adjusted coordinates, calculated according to variants 1-10, in reference to the results obtained on the assumption that references points 10, 20 and 30 are zero standard deviation.

Table 5. Summary of differences between the coordinates adjusted
and obtained on the assumption that references points 10, 20, 30 are zero standard deviation [mm]

Coordinate/Variant	1	2	3	4	5	6	7	8	9	10
x10	0,0	0,0	0,0	-49,0	0,0	0,0	-62,6	-11,2	-17,5	-11,0
y10	0,0	0,0	0,0	42,3	0,0	0,0	0,0	5,5	15,7	18,4
x20	0,0	0,0	59,2	0,0	0,0	62,6	0,0	13,0	31,6	36,0
y20	0,0	0,0	-12,0	0,0	-214,9	0,0	0,0	-4,0	-19,0	-31,9
x30	0,0	-76,2	0,0	0,0	53,2	-71,2	-133,8	-1,8	-14,1	-25,0
y30	0,0	41,3	0,0	0,0	-73,0	37,5	37,5	-1,5	3,3	13,4
x1	0,0	-1,2	27,7	-18,2	58,8	24,0	-38,5	1,4	11,2	21,0
y1	0,0	-2,5	-19,8	12,1	-76,4	-42,7	-20,2	-4,2	-11,7	-14,2
x2	0,0	1,8	31,5	-3,9	136,5	26,7	-35,8	4,0	23,8	40,0
y2	0,0	2,3	-12,4	32,5	29,0	-13,9	-13,9	0,5	7,2	13,5

PRESENTATION OF THE RESULTS OF CALCULATIONS

On the basis of calculations, assuming zero standard deviation for points to establish, in Fig. 1 and 2 there are shown a constant probability density ellipse, demonstrating the differences between the positions of points designated for different way to establish. Similar sketches are presented for the case taking into account the inaccuracy of references points (fig. 3 and 4).

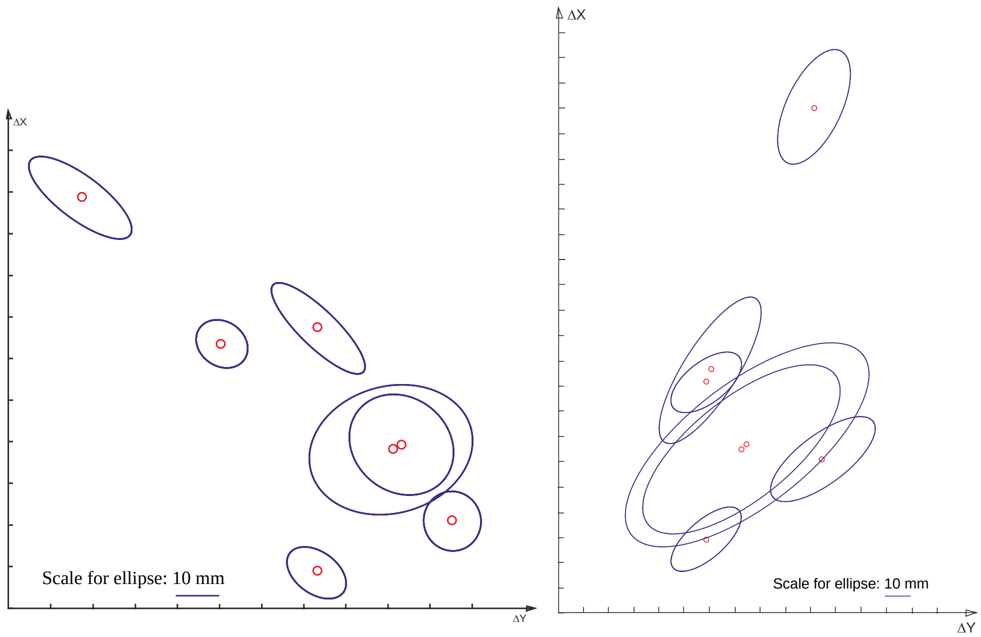

Figure 1. Illustration of differences in the position of points 1 and 2 and zero standard deviation ellipses for point 1 and 2 depending on the method of establishing

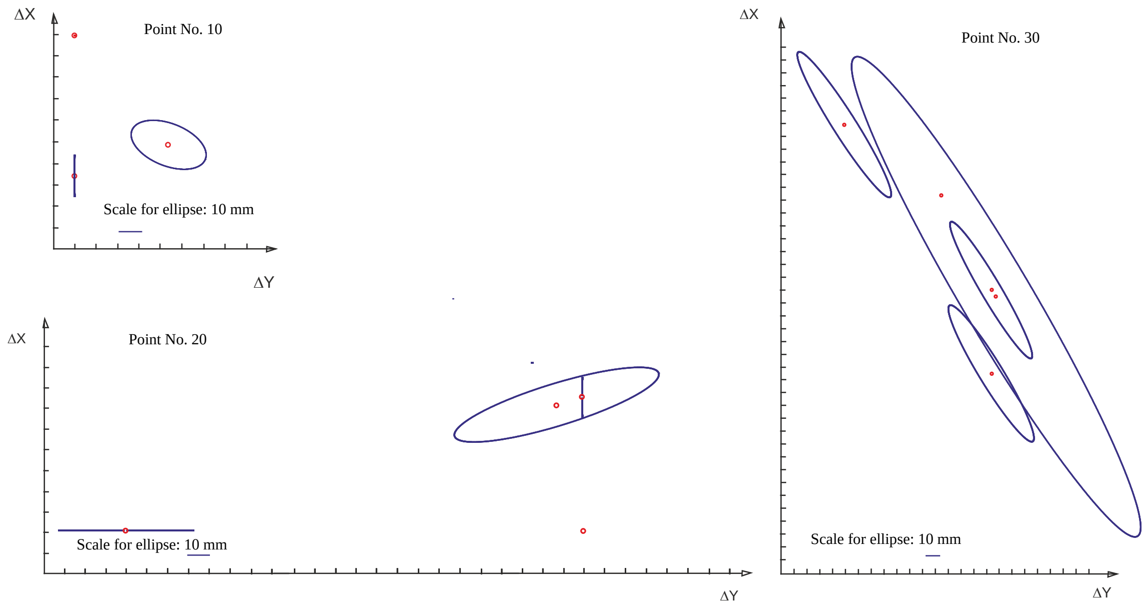

Figure 2. Illustration of the difference in location points 10, 20, 30 and the shape of the zero standard deviation ellipses for these points, depending on the method of establishing

It should be noted that the designated ellipses of constant probability density depicted in the diagrams are set at a probability level of P = 0.39. However, it is easy to see, that even a significant increase in the probability (based on the χ^2 distribution or the F-Snedecore distribution) will not provide what should guarantee the strict adjustment – coverage by ellipses at a predetermined probability level of the position of the designated points of the network.

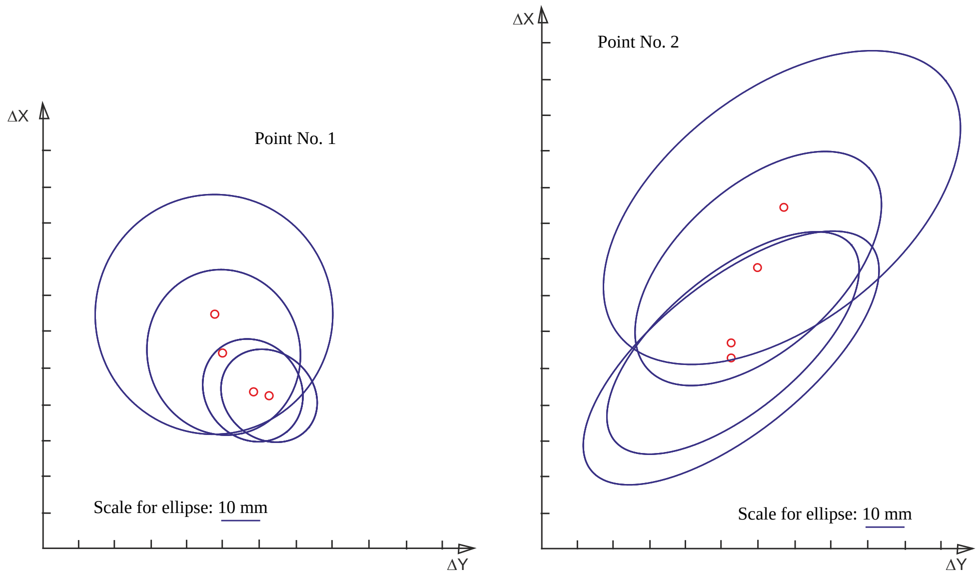

Figure 3. Illustration of differences in the position of points number 1 and 2 and ellipses of constant probability of point 1 and 2, depending on the accuracy of references points (zero standard deviation, 1, 5, 10 cm)

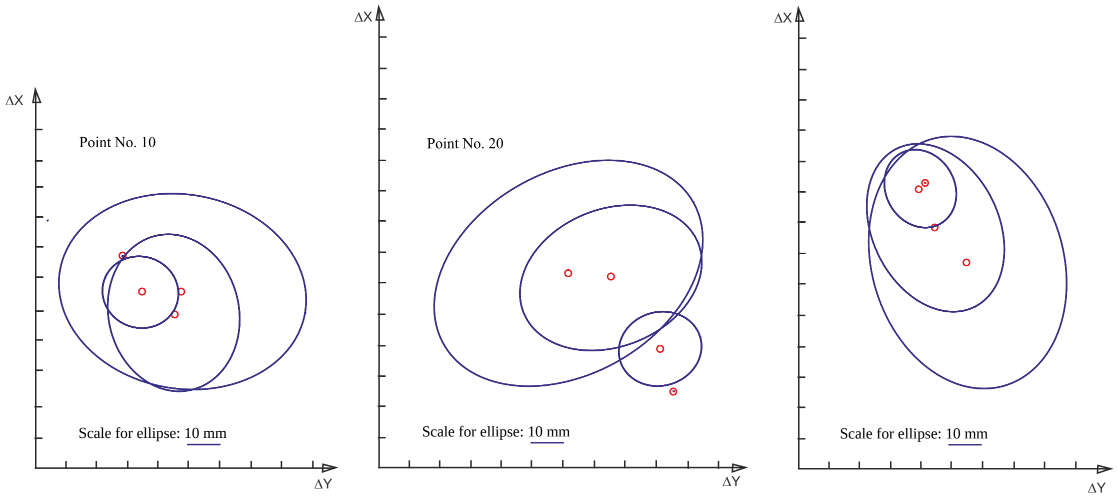

Figure 4. Illustration of the position differences of point number 2 and ellipses of constant probability for points 10, 20, 30, depending on the accuracy of references points (error-free, 1, 5, 10 cm)

CONCLUSION

Analyzing the calculations carried out by variants 1-7 and 8-10, it is worth paying attention to significant discrepancies between their results. These disparireferencess relate to both estimators of coordinates, as well as their parameters of assessing the accuracy. They are reflected particularly in the case of the adoption of a different approach to get references points as zero standard deviations (Figs. 1 and 2). For the case in which it is assumed the fallacy of references points (including the case of equal variance for the respective coordinate points), differences in the final results are also visible, but achieve significantly lower level (Figs. 3 and 4).

In each of the cases, adjustment of network was carried out strictly, using he method of least squares. At this point, there is an elementary question. How to understand and interpret the concept of strict adjustment? According to a pragmatic approach, it should be carried out in such a way as to obtain always the same values of estimators. Meanwhile, the concept of strict network adjustment amounts, de facto, only to apply strict rules of mathematical statistics, which do not include either way of geodetic connection or that the standard deviations of references coordinates are not actually equal zero .

To give the process of adjustment the strict nature, it is necessary to include full variance-covariance matrix for the references points. Of course, the choice of references points will have some influence on the final results, but this will impact negligible in comparison with the assumption of correctness establish points.

ACKNOWLEDGMENTS

This research was financed by the Department of Geomatics of the AGH University of Science and Technology in Kraków statutory research funds.

REFERENCES

[1] Czaja J., Preweda E.: Analiza statystyczna zmiennej losowej wielowymiarowej w aspekcie korelacji i predykcji, [*Statistical analysis of multidimensional random variable considering correlation and prediction*], Geodezja, T.6, z.2, AGH, Kraków, pp.129–145, 2000.

[2] Hanus P., Jasińska E., Preweda E.: Analysis of the accuracy of determining the coordinates property borders. International Conference on Environmental Engineering, Vilnius, pp.1-7, 2014

[3] Jasińska E., Preweda E., Estimation linear model using block generalized inverse of a matrix. International Multidisciplinary Scientific GeoConference SGEM, vol. 2, Sofia, pp. 167-174, 2013

[4] Jasińska E., Preweda E.: Effect the accuracy of benchmarks to establish of the determination of geodetic network. International Conference on Environmental Engineering, Vilnius, pp. 1-8, 2014

[5] Jasińska E., Preweda E.: Wybrane problemy wyrównania sieci geodezyjnych [*Selected problems of geodetic network adjustment*], AGH, Kraków, 2016 (w druku).

[6] Latoś S., Maślanka J., Preweda E.: Analiza dokładności poziomych osnów geodezyjnych zakładanych metodą poligonową z wykorzystaniem tachimetrów elektronicznych. Geodezja, 3, AGH, Kraków, 1997

[7] Latoś S. Preweda E.: Możliwości i efekty stosowania tachimetrów elektronicznych do zakładania osnów realizacyjnych dla specjalnych celów. Nowoczesna technika i technologia geodezyjna w praktyce, GEA'89, Katowice, 1998

[8] Latoś S., Preweda E.: Ustalenie kryteriów dokładnościowych wyznaczania położenia punktów poziomej osnowy pomiarowej przy wykorzystaniu tachymetrów elektronicznych. Geodezja, T.5, z.1, AGH, pp. 55–71, Kraków, 1999.

[9] Preweda E.: Baza informacji na temat przemieszczeń poziomych terenu. ZN AR, XIV, 324, Geodezja i Urządzenia Rolne, Wrocław 1997

[10] Preweda E.: Estymacja parametrów kinematycznego modelu przemieszczeń, [*Estimation of parameters of deformations kinematics model*], AGH, Kraków, p. 101, 2002.

[11] Preweda E.: Modelowanie informacji o obiektach inżynierskich w aspekcie wyznaczania przemieszczeń i odkształceń. Geodezja, T.9, z.2/3, AGH, Kraków, pp 811-834, 2003

[12] Preweda E.: Outlier detection in surveying networks. International Multidisciplinary Scientific GeoConference SGEM, Albena, pp. 365–372, 2014

[13] Preweda E.: Rachunek wyrównawczy ⇒ modele statystyczne [*Adjustment computations ⇒ statistical models*], Progres, Kraków, p.387, 2013

[14] Rozporządzenie Ministra Administracji i Cyfryzacji z dnia 14 lutego 2012 r. w sprawie osnów geodezyjnych, grawimetrycznych i magnetycznych. Dz. U. z dnia 30 marca 2012 r. poz. 352.