Transformations of qubit system states within quantum circuits after passing through Hadamard gates

Swierczewski Lukasz[1)], lswierczewski@pwsip.edu.pl

[1)]Computer Science and Automation Institute, College of Computer Science and Business Administration in Lomza

PWSIiP
The State College of Computer Science and Business Administration in Lomza

Abstract

The thesis presents the possible implementation concerning the realization of multi-qubit Hadamard gates, which constitute one of the most commonly used constituents during construction of quantum circuits. Elaboration of the model was based on the use of parallel computer programming with memory dissipated within the MPI environment.

Key words: *quantum calculations, parallel programming, quantum mechanics*

Motivation

Willingness to construct the model enabling a simulation of quantum gates, which differ significantly from their classical contemporaries.

Theoretical bases

Qubit, unlike the bit does not need to have a clearly determined status, and it may assume the given value only with a determined probability.

Classic qubit definition is as follows:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where α and β values are determined by complex numbers known as amplitudes, and their squares represent the probability of the fact that qubit is in the given status.

Therefore the statement on complementing the sum of amplitude squares (and simultaneously the likelihood) to one must be true:

$$|\alpha|^2 + |\beta|^2 = 1$$

As far as literature is concerned, Hadamard's gate is usually marked with the symbol H and for the size of one qubit it is presented as follows:

$$\mathrm{H} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

It may be defined in a recurrent manner by the following formula:

$$\mathrm{H}_m = \frac{1}{\sqrt{2}}\begin{bmatrix} H_{m-1} & H_{m-1} \\ H_{m-1} & -H_{m-1} \end{bmatrix}$$

$$\mathrm{H}_0 = +1$$

$$\mathrm{H}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\mathrm{H}_2 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

or in iterative manner as:

$$(\mathrm{H}_n)_{i,j} = \frac{1}{2^{n/2}}(-1)^{i \cdot j}$$

where i · j is the bitwise dot product of the binary representations of the numbers i and j.

Implementation

<u>Step 0. Start</u>
Step 1. Enter N
Step 2. Enter or generate the input vector determining the status of N qubits
Step 3. Initiate Hadamard's matrix for the gate with N size within the dispersed system

Step 4. I = 1

Step 5. Until I is smaller or equals $\sqrt{N}$ go to Step 6 otherwise go to Step 8

Step 6. Subvector number I send to all processes from I · $\sqrt{N}$ +1 to (I+1) · $\sqrt{N}$ -1
Step 7. Increase I by one

Step 8. Perform multiplication of submatrix by subvector within each process, and record the resulting vector in a subvector.

Step 9. I = 1

Step 10. Until I is smaller or $\sqrt{N}$ equals go to Step 11 otherwise go to Step 13

Step 11. Perform reduction to the sum of subvectors from I · $\sqrt{N}$ +1 to (I+1) · $\sqrt{N}$ -1, and enter the result to subvector I of the root process
Step 12. Increase I by one

Step 13. Return the value of root process subvectors
<u>Step 14. Exit</u>

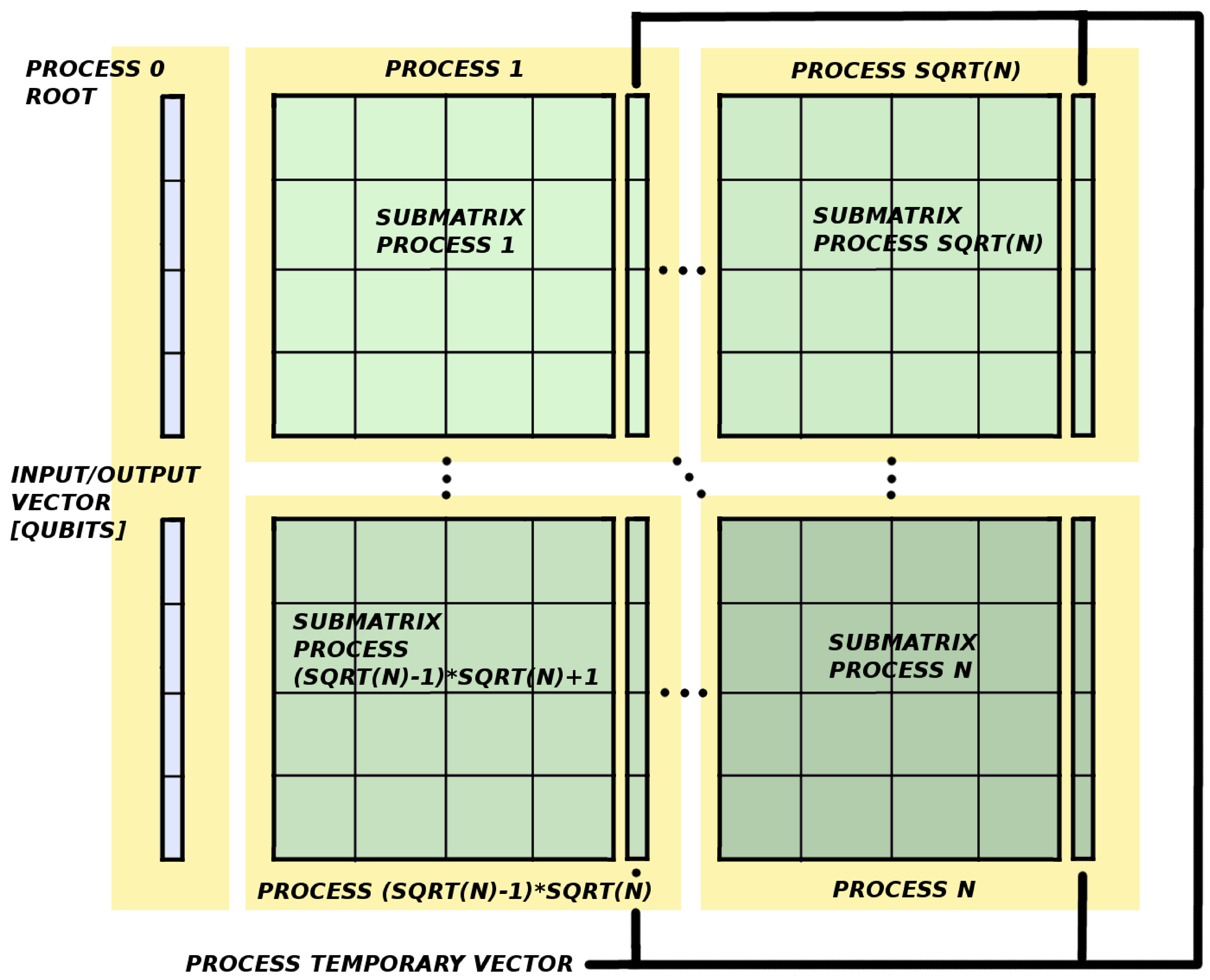

Figure 1: Structure of the scattering data in the calculation model

Summary

Creation of systems from quantum gates is not as obvious as in case of classic gates, where we always obtained sequences of zeros and ones on outputs and inputs. The sole design of quantum algorithms is also not equally intuitive. Simulating quantum calculations on classic computers is related with numerous problems – it has been proved that it is impossible to effectively perform simulation of quantum devices with the use of modern technology.

Literature

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Note

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