

dr hab. inż. Witold Pytel<sup>1), 2)</sup>  
mgr inż. Bogumiła Pałac-Walko<sup>2)</sup>

Recenzent: prof. dr hab. inż. Jan Butra

## Strength of Weak Floor Strata Beneath Rock Pillars

*Keywords: weak floor strata, ultimate bearing capacity, approximate approach, finite difference method*

### **Abstract**

*A simplified method for stratified floor strata ultimate bearing capacity (UBC) assessing, in room-and-pillar mining conditions, is presented. The physical problem consists of the following two different cases of rectangular rigid pillar of size  $B \times L$  resting on a two-layer floor system: (a) with the upper weaker layer of thickness  $H$  overlying a stronger, infinite rock deposit. It has been assumed that bearing capacity of floor strata may be analyzed as a shallow foundation problem, using the general bearing capacity equation given by Brinch Hansen with appropriate shape, inclination, and surcharge depth factors, or may be treated as a punching failure problem, particularly in a case of relatively strong upper stratum presence. A broad review of available analytical techniques did show that a finite thickness of the upper weaker layer causes a significant increment/decrement in its ultimate bearing capacity in comparison with homogeneous conditions. The proposed approach has been also validated by numerical modeling utilizing FLAC3D the finite difference computer code.*

### **Introduction**

Presently, design of hard rock pillars for weak floor strata conditions is based on the weak floor strata ultimate bearing capacity (UBC). Design techniques utilized here typically estimate UBC underneath full size pillars using Vesic [1] analysis which considers weak floor strata as a two-layer cohesive soil or rock system, with a weak layer overlying a stiffer layer and the angle of internal friction ( $\phi$ ) for both layers equaling to zero. The cohesion ( $c$ ) of the weak layer is estimated more often from its moisture content [2] than from the laboratory compressive strength data as proposed by Vesic. This technique, called Vesic-Speck approach, is used most commonly for mining applications in the U.S.A. today; however, this analysis technique has never been validated for shallow foundations on layered rock strata involving a weak layer(s) overlying a stronger rock layer. To overcome

<sup>1)</sup> KGHM CUPRUM sp. z o.o. CBR, ul. gen. Wł. Sikorskiego 2-8, 53-659 Wrocław

<sup>2)</sup> Politechnika Wrocławska, Wydział Geoinżynierii, Górnictwa i Geologii,  
Plac Teatralny 2, 50-051 Wrocław

most of those limitations, the analytical Pytel-Chugh's approach has been developed [3], including the effect of adjacent pillars and the non-zero values of the angle of internal friction for both layers. Since then this technique has been successfully applied mainly in Midwestern coal mines in U.S.A.

It has been assumed that bearing capacity of coal pillars on floor strata may be analyzed as a shallow foundation problem, assuming the validity of the foundation general bearing capacity. All field observations show that a finite thickness of the upper weaker layer causes a significant difference in its ultimate bearing capacity in comparison with homogeneous conditions. For a purely cohesive floor system ( $\phi=0^\circ$ ), the Vesic's approach is most useful and convenient; but where cohesion and the angle of internal friction for weak floor strata can not be assumed to be equal to zero, the Mandel and Salencon [4] single-layer technique, based on the slip-line method with Coulomb criterion of failure is more suitable.

Figure 1 depicts the problem where multiple pillars of width  $B$  and length  $L$ , spaced  $s$  apart, are resting on a finite layer of weak floor stratum ( $c_1, \phi_1, \gamma_1$ ) of thickness  $H$  underlain by a deformable infinite layer with parameters  $c_2, \phi_2, \gamma_2$ .

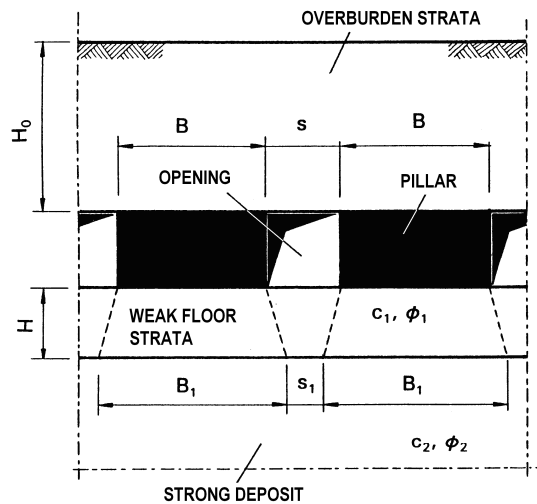


Fig. 1. Physical Problem

It should be noted that currently updated Pytel-Chugh's method is the only technique which considers the effect of adjacent pillars and the angle of internal friction not equal to zero for both floor deformable layers. To date, the appropriateness of this design technique has been also established through field observations in Polish deep copper mines [5]. This approach may be summarized as follows:

- the foundation is considered as consisting of two layers, with a weak floor layer overlying the stiff layer,

- the effect of adjacent pillars and non-zero values of the angle of internal friction for both layers is included,
- a new modified bearing capacity factor  $N_t$  is calculated based on a combination of two theoretical solutions: the Mandel-Salencon [4] bearing capacity estimation approach (effect of finite thickness of the upper stratum), and the Mandel [6] approach (effect of the pillars' spacing).

### 1. The Proposed Approach

The ultimate bearing capacity of two-layer floor system  $q$  (Fig. 2) may be expressed by the following basic approximation:

$$\frac{1}{q} = \frac{1}{q_1} + \frac{1}{q_2}, \text{ or } q = \frac{q_1 q_2}{q_1 + q_2}, \quad (q_1 \leq q_2) \quad (1)$$

with the boundary conditions:

$$\begin{aligned} q &\rightarrow q_1 \text{ if } q_2 \rightarrow \infty \\ q &\rightarrow q_2 \text{ if } q_1 \rightarrow \infty \text{ (} H \rightarrow 0 \text{)} \\ q &\rightarrow 0 \text{ if } q_1 \rightarrow 0 \end{aligned}$$

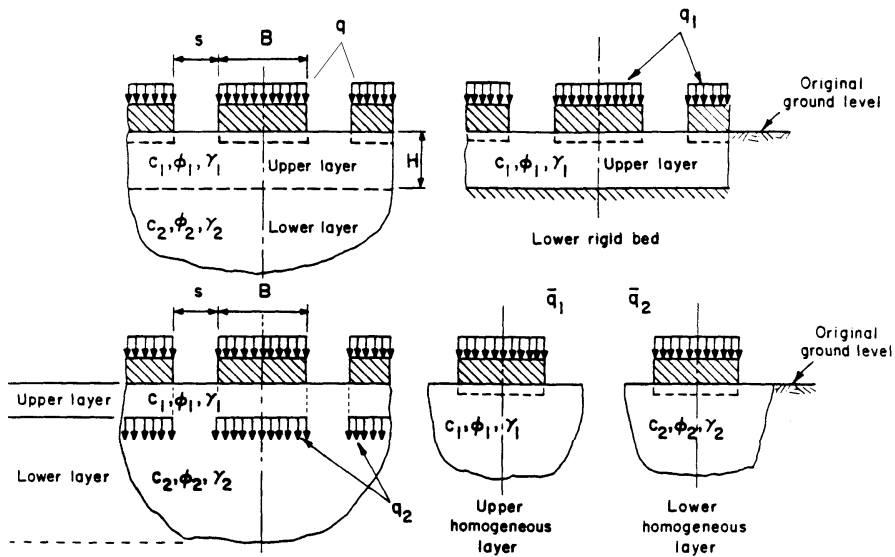


Fig. 2. Proposed approach for calculation of pillar bearing capacity

where:  $q_1 = \bar{q}_1 \frac{F_{c1}}{\bar{F}_{c1}}$  represents the UBC of pillar substrata, assuming the lower layer to be infinitely rigid ( $F_{c1}, \bar{F}_{c2}$  Mandel & Salencon coefficients, Fig. 3) and spacing  $s$  between pillars is greater than:

$$s_0 = 2 \frac{H}{\cos \phi_1} \cos\left(\frac{\pi}{4} - \frac{\phi_1}{2}\right) \exp\left[\left(\frac{\pi}{4} - \frac{\phi_1}{2}\right) \tan \phi_1\right] \quad (2)$$

$q_2 = \xi \bar{q}_2$  represents bearing capacity of the lower layer including factor of majoration:  $\xi\left(\frac{S}{B}, \phi_2\right)$  due to the presence of adjacent pillars [6], (see Fig. 4), while  $\bar{q}_1, \bar{q}_2$  are the ultimate bearing capacities of the floor strata composed of layers 1 or 2 exclusively (homogeneous conditions), subjected to the load transmitted from the pillar:

$$\bar{q}_1 = c_1 N_{c1} s_c \bar{F}_{c1} + \frac{1}{2} \gamma_1 B N_{\gamma1} s_{\gamma} \bar{F}_{\gamma1}, \quad \bar{q}_2 = c_2 N_{c2} s_c \bar{F}_{c2} + \frac{1}{2} \gamma_2 B N_{\gamma2} s_{\gamma} \bar{F}_{\gamma2}$$

where:  $\bar{F}_c$  and  $\bar{F}_\gamma$  are the Mandel & Salencon's coefficients for homogeneous conditions for a given value of angle of internal friction  $\phi$  (for  $\phi = 0^\circ - \bar{F}_c = 0.778$ , for  $\phi = 10^\circ - \bar{F}_c = 0.691$ ; for  $\phi = 20^\circ, \bar{F}_c = 0.585$ ; for  $\phi = 30^\circ, \bar{F}_c = 0.460$ ),  $N_c$  and  $N_\gamma$  are bearing capacity coefficients and  $s_c$  are shape factors.

Actually there is no real physical basis for Equation 1. It is a kind of formal tool which couples two separate analytical solutions since there is no closed expression for the general two-layer floor problem. The proposed form of Eq. 1 is the simplest one which satisfies exactly the mentioned boundary conditions, however for finite values of  $q_1$  and  $q_2$  it produces an error which can not be determined *a priori*. Therefore this equation has been transformed into the following form:

$$q = \frac{q_1 q_2}{q_1 + q_2} + E_r \frac{\bar{q}_1}{q_2}, \quad (q_1 \leq q_2) \quad (3)$$

where  $E_r$  is the maximum error which occurs when  $\bar{q}_1 = \bar{q}_2$ . From Eq. 3 one may obtain:

$$E_r = \frac{\bar{q}_2}{\bar{q}_1} \left[ q - \frac{q_1 q_2}{q_1 + q_2} \right] \quad (4)$$

The error term is assumed to have the form  $E_r \frac{\bar{q}_1}{\bar{q}_2}$  because it is able to express conveniently the deviation from the exact value in function of difference between two layers' strength parameters. If  $\bar{q}_1 = \bar{q}_2$  then  $c_1 = c_2$  and  $\phi_1 = \phi_2$  and then two-layer system transforms into one-layer system (independently on H/B) with row of pillars on the boundary surface and then Eq. 4 may be simplified into the following form:

$$E_r = \bar{q}_1 \left[ \xi - \frac{1}{\frac{1}{\xi} + \frac{1}{F^*}} \right] \quad (5)$$

where:  $F^* = \frac{F_c}{F_c}$ ,  $F_c$  is the Mandel-Salencon coefficient (Mandel & Salencon, 1969) suitable for angle of friction between coal pillar and the upper floor stratum  $\delta = \frac{2}{3}\phi_1$  (values of  $F_c$  may be find in Figure 3),  $\xi$  is the Mandel factor of majoration (Mandel, 1965) calculated using a given distance  $s$  and angle  $\phi_2$  (values of  $\xi$  are presented in Figure 4). Substituting (5) to (1) and assuming a weightless material underneath the shallow pillar we finally obtain:

$$q = \bar{q}_1 N_t = \bar{q}_1 \left\{ \frac{1}{\frac{\bar{q}_1}{\bar{q}_2} \frac{1}{\xi} + \frac{1}{F^*}} + \frac{\bar{q}_1}{\bar{q}_2} \left[ \xi - \frac{1}{\frac{1}{\xi} + \frac{1}{F^*}} \right] \right\} \quad (6)$$

## 2. Numerical Example

Design approaches presented in the paper are illustrated with a physical problem solution with the following geometry:

- ratio of pillar length over its width:  $L/B = 100$ ,
- ratio of opening width over pillar width:  $s/B = 4.0$  (for results shown in Fig. 5) and  $s/B = 0.5$  (Fig. 6)
- angle of internal friction for floor strata:  $\phi_2 = 40^\circ$ .

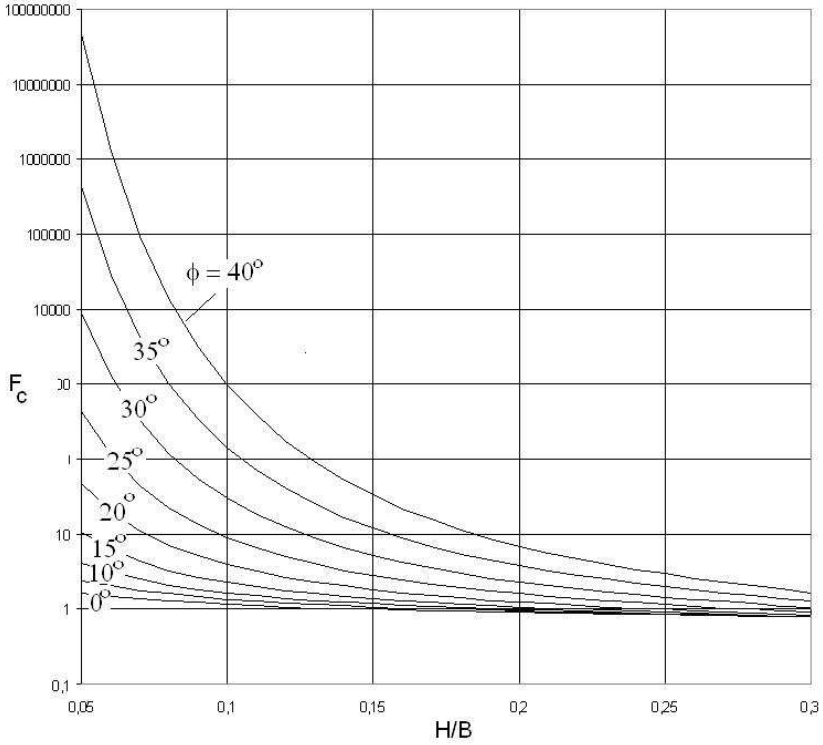


Fig. 3. Values of  $F_c$  of Mandel-Salencon (1969) Coefficient  $F_c$

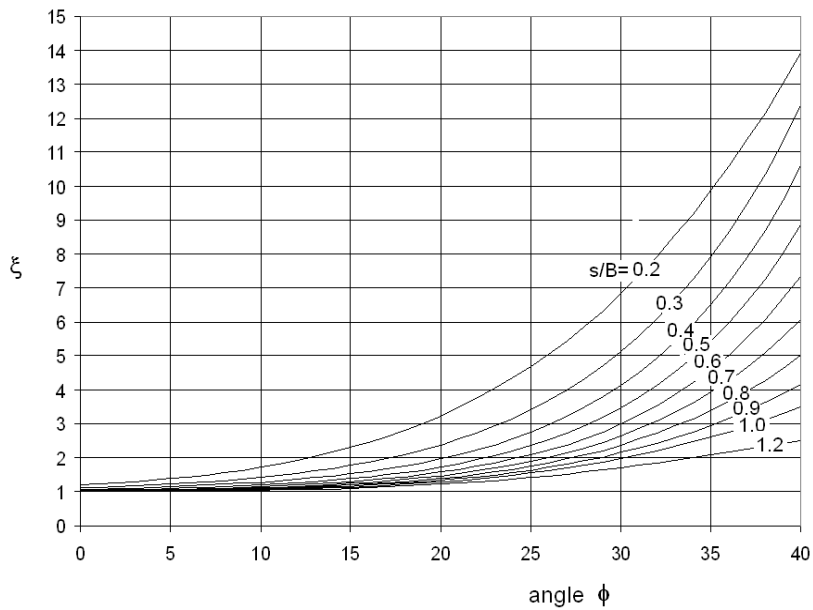


Fig. 4. Contour of Mandel (1965) Factor of Majoration  $\xi$ .

This physical problem was also analyzed using different values for relative floor strata thickness  $H/B$  and the ratio  $\frac{\bar{q}_1}{\bar{q}_2}$ . Calculation results are presented in Figures 5-6.

Differences between results obtained using the proposed approximate approach and the finite difference method (FDM) are presented in Fig. 7 for a chosen case of underground geometry and geotechnical conditions.

### Conclusions

1. The updated analytical expression for UBC of immediate two-layer weak floor strata (Eq. 6) is more realistic than its previous version since it has been associated in some instances (e.g. for very low  $H/B$  and high values of  $\phi_2$  or for  $\frac{\bar{q}_1}{\bar{q}_2}$  close to 1.0) with the unreasonable estimates of  $q$  greater than  $q_2$ .
2. The effect of pillar spacing is very important particularly for lower values of  $H/B$ .
3. The effect of  $\frac{\bar{q}_1}{\bar{q}_2}$  ratio on the resultant value of ultimate bearing capacity  $q$  is almost linear in a case where the lower, stronger stratum is located at the moderate depth ( $H/B \cong 0.2$ )  $\rightarrow q = \frac{\bar{q}_1}{\bar{q}_2} q_2$ .
4. The updated estimate of immediate floor strata bearing capacity has been proved to be a very convenient and relatively accurate analytical tool for room-and-vent pillar systems design.

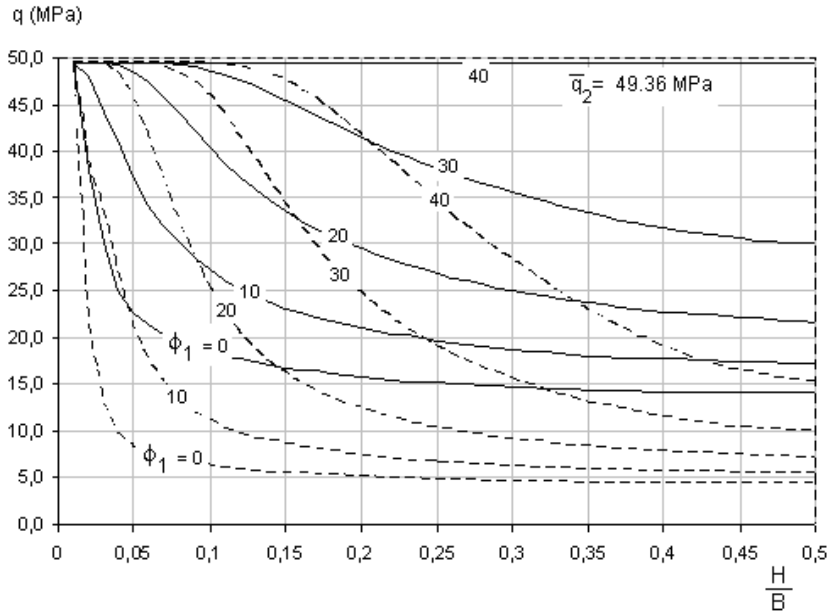


Fig. 5. The effect of  $H/B$  on two-layer immediate floor strata UBC  
 $c_1 = 0.5$  MPa (---),  $c_1 = 2.0$  MPa (—),  $c_2 = 2.0$  MPa,  $\phi_2 = 40^\circ$ ,  $s/B = 4.0$

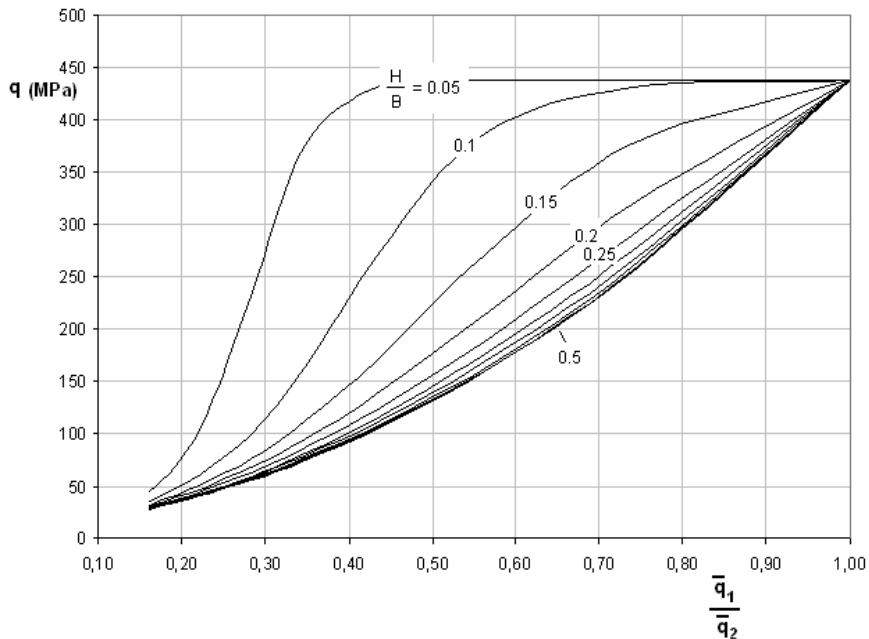


Fig. 6. The effect of  $\frac{\bar{q}_1}{\bar{q}_2}$  ratio on UBC of immediate floor strata

$c_1 = c_2 = 2.0$  MPa,  $\phi_2 = 40^\circ$ ,  $s/B = 0.5$



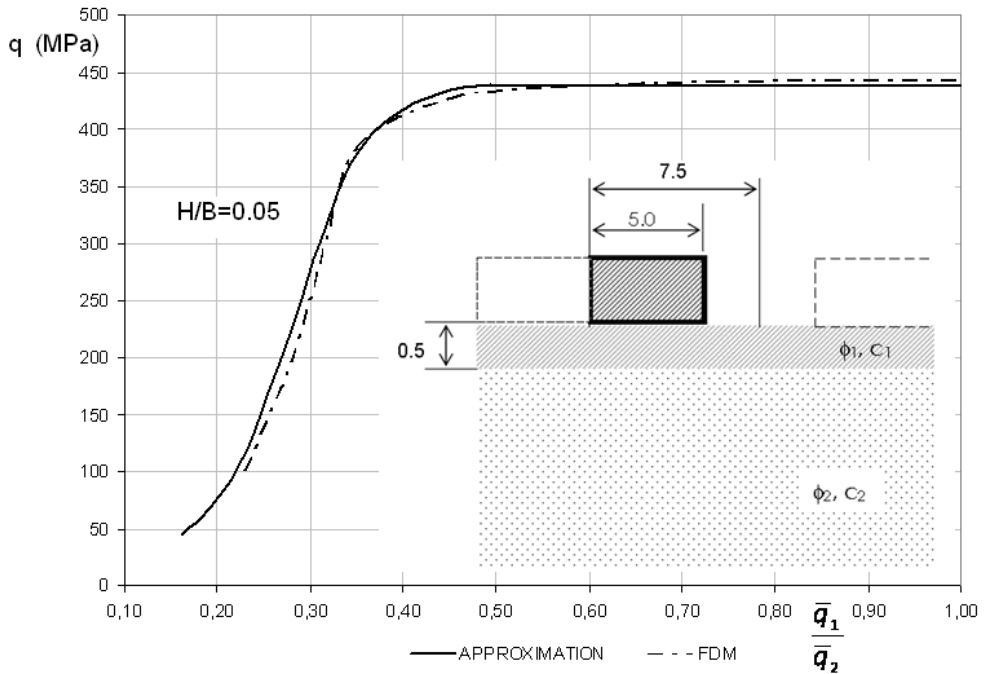


Fig. 7. The effect of  $\frac{\bar{q}_1}{\bar{q}_2}$  on two-layer immediate floor strata UBC determined using the proposed approximate approach and the finite difference method (Flac3D)  
 $H/B=0.05$ ,  $c_2 = 2.0$  MPa,  $\phi_2 = 40^\circ$ ,  $s/B = 0,5$

### References

- [1] Vesic A.S., 1970, Research on Bearing Capacity of Soils (unpublished).
- [2] Speck R.C., 1981, The influence of Certain Geologic and Geotechnical Factors on Coal Mine Floor Stability – Case Study. Proc. 1st Int. Conf. on Ground Control in Mining, Morgantown, WV.
- [3] Chugh Y.P. and Pytel W., 1992, Design of Partial Extraction Coal Mine Layouts for Weak Floor Strata Conditions. U.S.B.M. IC 9315, s. 32-49.
- [4] Mandel J. and Salencon J. (1969). Force Portante d'un Sol Sur une Assise Rigide. Proc. tth ICMFE, Mexico City, 2, s. 157-164.
- [5] Pytel W., 2002, 2003, The Actual Characteristics of pillar-Floor System Interaction Based on Pillar Deformation Measurements (in Polish). Stage 1-2, Res. Rep. CBPM Cuprum, Wroclaw (unpublished).
- [6] Mandel J. Interference Plastique de Semmelles Filantes. Proc. 6th ICMFE, Montreal, 3, s. 127-131.

## **Wytrzymałość spągu uwarstwionego obciążonego pod filarem**

*Słowa kluczowe: słaba warstwa spągowa, opór graniczny podłoża, metoda różnic skończonych*

*W artykule przedstawiono uproszczoną metodę szacowania nośności uwarstwionego spągu, dla warunków eksploatacji metodą filarowo-komorową. Model fizyczny zagadnienia opisuje przypadek prostokątnego, sztywnego filara o wymiarach  $B \times L$  spoczywającego na dwuwarstwowym układzie spągu bezpośredniego, ze słabszą warstwą położoną wyżej o grubości  $H$ , zalegającą nad mocniejszą warstwą skalną o nieskończonej miąższości. Przyjęto, że nośność podłoża może być analizowana jako przypadek fundamentu bezpośredniego poprzez ogólne równanie nośności zaproponowane przez Brincha Hansena, z zachowaniem odpowiedniego kształtu, nachylenia oraz współczynnika zagłębienia. Przegląd dostępnych matematycznych analiz stosowanych dla opisanego problemu wskazuje, że skończona miąższość słabszej warstwy powoduje wzrost/spadek wartości nośności, w porównaniu z warunkami panującymi w górotworze jednorodnym. Proponowane podejście rozwiązania problemu zostało zweryfikowane przy zastosowaniu metod numerycznych wykorzystujących metodę różnic skończonych (FLAC3D).*