EDUKACJA HUMANISTYCZNA nr 2 (29), 2013
Szczecin 2013

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SOLVING PLOT MATHEMATICAL PROBLEMS AS THE WAY TO ACQUIRE KEY AND SUBJECT COMPETENCIES BY SCHOOLCHILDREN

The system of education in Ukraine along with most European countries is being reconstructed on the basis of competency-based education that provides the standardization of educational results and forming the competencies that help young people to act quickly in our rapidly changing world and to respond promptly to the changes of living environment. The competency is considered as the ability, willingness and readiness of a person to solve typical and problematic situations that occur in casual and professional sphere.

Key competencies determine the successful adaptation of a human in the society, and though we still do not have a generally accepted definition of key competencies, they are mostly understood as a person's ability to cope with various tasks. Key competencies (OECD) are used to determine the list of competencies that allow a person to take part effectively in many social spheres and that contribute both to improving the quality of our society and to achieving personal success that can be applied in many life spheres. Key competencies form the basic number of most general concepts that should be organized into the complex of knowledge, abilities, skills, values and attitudes according to the educational sectors and areas of pupils' school life[1].

During the international project "Definition and Selection of Key Competencies", implemented by the Organization for Economic Cooperation and Development and by National Institutes for Education Statistics in Switzerland and the U.S. there was not determined a final definition of key competencies. During the symposium of Council of Europe "Key Competencies for Europe" there was proposed a list of key competencies: learn, search, think, work with, get to work and adapt oneself[2].

The possession of key competencies let a person be successful in any sphere of professional and public activity, including also private life. Key competencies help to

[1] Овчарук О.В., *Розвиток компетентнісного підходу: стратегічні орієнтири міжнародної спільноти // Компетентніцстний підхід у сучасній освіті: світовий досвід та українські перспективи: бібліотека освітньої політики* / Під заг. ред. О.В.Овчарук. – К.: „К.І.С.", 2004. – С.11.

[2] Hutmacher W., *Key competencies for Europe* // Report of the Symposium Berne, Switzerland 27-30 March, 1996. Council for Cultural Co-operation (CDCC) a // Secondary Education for Europe Strasburg. 1997.

get positive results in uncertain and problematic situations. Moreover, the competency "learn" is considered as: to be able to benefit from the experience; to organize the relationship of knowledge and ability to put them in order; to organize personal study methods; to be able to solve the problems; to be able to study alone. That is why, the obtaining of key competencies may be implemented under the terms of uncertainty, under the process of problem solving; by the way, plot mathematical problem may be considered as the problem itself.

The authors of the international project "Definition and Selection of Key Competencies" consider the following competencies to be key: reflexive autonomous action, interactive use of tools, participating in the heterogeneous groups, critical thinking, solving the problems[3]. The plot mathematical problems are the models of real life situations, that is why solving them is a simulator that helps to obtain the ability to operate effectively in a rapidly changing world.

We consider a plot problem as a mathematical problem with some life story that is a quantitative side of the real processes, phenomena and situations and that contains the requirement to find the unknown value on the basis of the values given and relations between these values.

Taking into consideration that plot mathematical problems are the means of forming key competencies among schoolchildren, their solving have the following objectives:

1. Forming common approach, common skills and abilities of solving any problems;
2. A deep knowledge and mastering of the mathematical concepts that are being studied and the general concepts of science and everyday life;
3. Mastering the concepts of model, simulation and mathematical simulation; development of thinking, sharpness in pupils and their creative potential.

Solving a problem is a complex process of human mental activity that aims to transform the object into the resolving of conflicts between the terms and requirements of a problem. That is why a person has to analyze the basic data of a given situation all the time, to plan various options for its solution, to predict the consequences of particular actions on the process of solving as a whole, to choose the optimum, to perform the selected actions and to control this process; and this is not the end of solving a problem. A person must comprehend his/her solution in order to analyze the obtained results and to find another ways of solving such kind of problems.

During the life time a person always faces different everyday, professional and private problems. Pupils get the first experience of solving them at the Elementary School on the basis of plot mathematical problems. The process of solving plot problems is considered to be a "conversion" of a verbally given plot that contains numeric components and typical structure into the language of the arithmetical record, like a transition from verbal models to mathematical or schematic ones. The basis of this transition is text analysis and selection of mathematical concepts and relations.

Solving activities can be carried out using both algorithmic and heuristic ways. If a pupil fulfills the requirements, in such a case an algorithmic way of solving problems

[3] Definition and Selection of Competencies. Theoretical and Conceptual Foundations (DESECO). Strategy Paper on Key Competencies. An Overarching Frame of Reference for an Assessment and Research Program – OECD (Draft). – p. 8.

is being used. It means that a pupil performs his own activity in accordance with a well-known algorithm. Elementary steps consist in using the well-known algorithm of solving the given class of problems. But in order to do so, while analyzing the problem, a pupil must understand, to which class of problems a given problem belongs. The main difference between a heuristic and algorithmic way of problem solving is that a heuristic way does not have any algorithm. And, in such a case, the main task of a pupil is to find the plan or another means to solve the problem. In order to solve the non- algorithmic type of problems, a wide range of heuristic rules and schemes are being used. However, their implementation does not guarantee finding the elementary actions system that will lead to a complete solution of the problem.

So, if a pupil, at the beginning of a problem solving, does not have an approximate basis for his/her actions, he/she finds it out, carrying out the heuristic activity. Such kind of activity is carried out using special techniques – heuristics. First, heuristics are any means (graphic schemes, printed instructions, oral instructions of the teacher, visual materials, information etc.), that make it easier and more possible to solve a problem (M. Bulk, G. Bulk, G. Bull, K. Jung etc.) Second, heuristics are methods of solving certain classes of problems, that are not under strict algorithmization (V. Andreev, O. Yepisheva, V. Krupich, E. Skafa, Z. Slepkan etc.). And, at last, heuristics are specific mental techniques that compose the search strategy and tactics (A. Artyomov, N. Zilberberg, L. Larson, Y. Kolyagin, Y. Kulyutkin, G. Sarantsev, E. Skafa, L. Friedman etc.).

It is proved that the dominant heuristic, that is used at the Elementary School while solving plot mathematical problems, is a simulation- simulation of both problem situation (building support models, such as subject, schematic, verbal ones) and the process of solving it (schemes for analytical and synthetic analysis of the problem, "trees of considerations"), because simulation provides the necessary orientation in a problem situation.

Regardless of the method (algorithmic or heuristic) pupils' problem solving activity is the implementation of the main stages of solving by performing certain actions. Mostly Methodists define four stages of solving plot mathematical problems: 1. Familiarization with the problem. Analysis of the task's text. 2. Search for the ways of solving the problem. 3. The implementation of the plan of solving the problem. Recording solving and answers. 4. Work on the problem after it has been solved.

The last stage involves clarification of the fact that the obtained result satisfies the terms of the problem, checking the solving, analysis of solving, grounding the solving methods, consideration of another ways of solving, an investigation into the problem and its solving.

I. Familiarization with the problem. Analysis of the task's text. *To get familiar with*-that means, having read the problem formulation, to imagine a described life situation. *To analyze the problem's text* – that means to highlight the terms and questions; to identify the values the problem contains: known and unknown, to set relations between them. The problem analysis is carried out in two ways (after L. Friedman):

a) *subject-semantic analysis*-is a decoding the problem's terms in general, recovery of the real life situation, described in the problem. Generally such analysis is carried out orally, and the problem situation, produced on the basis of this analysis, creates a mental image of the plot of a problem;

b) logical-semantic analysis-aimed at identifying the peculiarities of the verbal set of separate values, both known and unknown, including required ones, and the most important- at identifying verbal signs of certain types of relations. This is the analysis of the task's text in order to set the values and relations between them, that are given in the task's text, thus breaking the task's text into certain elementary conditions (elementary condition of a problem is a statement, the task's text contains, that can not be divided into smaller statements) and requirements. Thus the structure of the problem appears.

As a result of a logical-semantic analysis of the task's text we can find out:

1) what values characterize the numerous side of those phenomena, processes and events, that the problem contains;
2) how many and what kind of values are given explicitly or implicitly in the task's text;
3) nature of every value: whether this value is known or unknown, and if it is unknown, then what is it – required, intermediate (auxiliary) or undefined;
4) what relations do these values have with one another;
5) what value is prevailing in every correlation, what word-signs, that are included in the value, point out the nature of this value;
6) what nature has each of these correlations (solvable, unsolvable); how are these correlations connected with one another.

Such analysis is possible only if fixation means are present – a model of the problem in the form of table, graph or picture. So, after the task's text analysis, we should carry out a gradual interpretation of the verbal model into the graphic (schematic) one and only afterwards-into the symbolic (mathematical) model.

Table, diagram, picture, drawing, that are the result of the problem's formulation, are the representative models of the problem. The construction of the representative model of the plot problem has got several goals:

a) to fix the results of the problem's analysis in order to organize this analysis. That is why the model's construction is being carried out in the process of analysis and its implementation;
b) to examine the problem from different angles. Construction of the problem's model makes it possible to carry out the most important thing that directs, encourages the process of solving and reformulating the problem;
c) construction of the problem's model is a preparatory stage for making up the problem's mathematical model[4].

Representative models may be subject, visual-schematic and tabular. The subject model of the plot problem is a visual recovery of the real situation, described in the problem. Such models may contain things, and may be given in the form of drawings, any kind of staging the problem's plot. This type of models includes also mental recovery of the real situation, described in the problem, in the form of images.

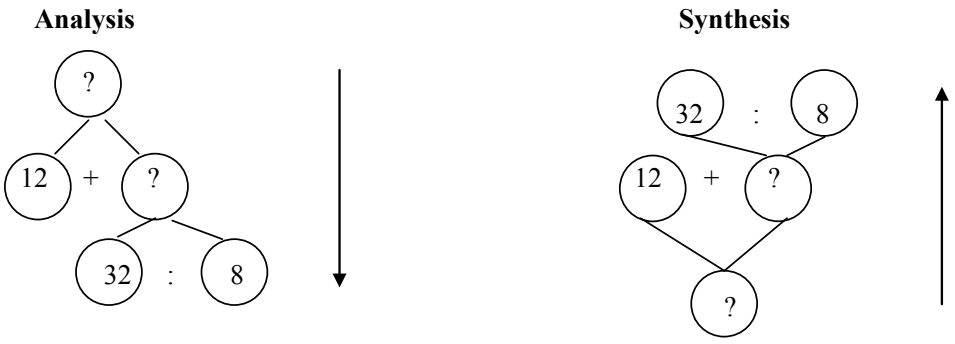
The visual-schematic models are used for generalized, schematic recovery of the problem's plot. This kind of models maintains visibility that is inherent in subject models, but they recover the real situation, described in the problem with the help of segments, geometric shapes etc. The visual-schematic models contain various schematic

[4] Фридман Л.М., *Сюжетные задачи по математике: История, теория, методика*. – М.: Школьная Пресса, 2002. – С. 95-99.

records of the problem's terms or short records in the form of a table, schematic drawings. Tabular form of a short record is used when there are several inter-related variables in the problem, each defined by one or more values.

We must pay our attention to one more kind of models – structural (graphs, schemes); they are used for visual demonstration of dependencies and relations between the given and required values that is for the visual demonstration of the mathematical structure of the problem solving. The construction of the problem's structural scheme may be attributed not only to the first stage of the work over the problem – that is to the task's text analysis, but also to the second stage-search for the problem's solving, as we clearly see the problem's solving model on the structural scheme.

II. Search for the ways of solving the problem. Search for the ways of solving the problem using arithmetical method can be implemented from the problem's questions to the numeric data – analytically, or from the numeric data of the problem to its questions- synthetically, for example:



These both methods are used in the practice of study, but synthetic method has its benefits; it is believed that synthesis is easier for schoolchildren. Meantime, usage of the synthetic method may cause additional difficulties-a child may choose numerical data at random, not paying attention to the fact, whether they have sense of solving the given problem. The analysis is mostly focused on making a plan of the problem solving. Here we should keep in mind not a single action, but reasoning in general, as we need certain numeric data, that may be unknown yet, to answer the problem's question. When analyzing, pupils are getting an idea of the problem solving in general, and not of separate steps that has been chosen, while the synthesis promotes processing ability to foresee, that can be found out with the help of two certain numeric data and directing schoolchildren' thoughts in the right direction.

One of the goals of problem solving is to form the ability of modeling the problem situations, including also making schematic drawings. Considering this, approach to the search for the ways of problem solving is slightly changing. The search, using subject or graphical model, when a child "reads" from schematic picture content and procedures that will lead to the problem solving is also possible.

For compound problems (when it is impossible to answer the problem's question using only one arithmetic operation) search for the problem solving ends at making a plan of solving, where the order of arithmetic operations is determined.

Having chosen one or another method or way of solving a plot problem, we should form the corresponding mathematical model for it. That means that, if an arithmetical method of solving is chosen, then the model is built as a computational formula or simply as the sequence of arithmetic operations (a plan of solving).

It should be noted that at the stage of searching for the ways of problem solving, we should pay special attention to the actions, which belong to the group of heuristic rules and schemes that are guiding the process of solving problems' activities.

III. The implementation of the plan of solving the problem. Recording solving and answers. Next the solving itself is being carried out: finding out the results of each planned arithmetic operation and estimation the content of a found value or finding the value of a numerical expression (expressions) - when using the arithmetical method of problem solving; solving the equation and answer to the problem's question- when using the algebraic method. So, the third stage of working over the problem is taking place.

IV. Work on the problem after it has been solved. Work on the problem after it has been solved means checking the correctness of solving.

To check the problem's solution is to decide whether it is correct or no, to be exact –if the obtained result satisfies the problem's terms. Checking the plot problems' solution may be direct or indirect and, in its turn, it may be complete or incomplete. A direct complete checking the problem means that we make sure that we have completed all the problem's terms, having found the unknown value; incomplete checking means that not all of the terms are being checked, but only some of them. Incomplete checking is done with the help of composing and solving the inverse problem.

1. Composing and solving the inverse problem. The inverse problem is composed by means of exchanging roles between one of the unknown values with any of the given values that means that the found value of one of the unknown values is taken for the given one, and one of the given values is considered as an unknown one. If, as a result of an inverse problem, we get the value, that coincides with the chosen given one, it means that the problem's solution is correct.
2. Solving the problem otherwise. We can carry out an indirect check by solving the problem otherwise. If a problem can be solved otherwise, the fact of obtaining similar results confirms that the problem is solved correctly. With a view to find out different ways of solving the problem it should be appropriate: redefinition of the problem's question; selection of auxiliary question; revealing the problem's hidden logical basis; visual arrangement of the problem.

At the Elementary School the ways of direct checking the solution's correctness are also used.

3. Finding the correspondence between the numbers, received as a result of a problem's solution, and the given numbers. When checking the problem's solution in such a way, arithmetic operations on the number that has been received as an answer to the problem's question are taking place: if we get the number, given in the problem's condition, then the problem is solved correctly. For example, solving the next problem: "Mother bought for the same price 3 kg of apples and 2 kg of pears. For all these products she paid

15grn. How much are the apples and the pears separately?", we found out, that the apples cost 9 grn and the pears – 6 grn.; having added the obtained numbers (9 + 6 = 15 grn), we get the number, given in the problem's condition. So, the problem is solved correctly.

4. An approximate estimation of an answer (finding the correspondence between the unknown number and the field of its values). This method means that before solving a problem, the field of an unknown number's values must be stated. That means that we must find out, whether the unknown number is smaller or larger than some of the given numbers. After the problem is solved, we must check if the obtained result corresponds to the stated field of values (in this case the problem may be solved correctly), or does not correspond (then the solution is wrong).This method helps to reveal the wrong solution and must be connected with another methods of checking.

Detection of the chosen solution's defects, search for the better solution, finding and fixing in the pupils' memory of those techniques and methods, that have been used in the solution, finding out the conditions of a possibility to use these techniques and methods- all of these factors help to transform the problem's solution into a powerful educational and training method. While discussing the process of solution, it is sometimes useful to find a possibility of generalization of this problem, to reveal its peculiarities, to compare the solution of this problem with the solved one before etc. That is why it is appropriate to use widely the problem's research by means of changing the situation, described in the problem; by means of changing the numeric data of the problem; by means of changing the unknown value and determining the impact of this change on the solution.

So, on the basis of work on the plot mathematical problems, a child is learning to analyze the situation comprehensively from childhood, to predict and determine the optimal operations to solve the problem. This way pupils are getting a key competence of solving the problems.

We should also pay our attention to the process of learning to solve problems. Based on the goal to form a key competence of coping with the difficulties, a mathematical problem should be considered as a problem, and learning to solve problems should be carried out by means of creating and solving the problem situations. It is possible by means of a thought-out method, when the problem of a new kind is obtained from the well-known among schoolchildren problem, by changing the conditions or questions, and determining the impact of this change on the solution, comparison of the solution of both kinds of problems and identifying common and distinctive features in them and finally determine the reasons for differences. This methodological approach can be applied when learning to solve any kind of plot mathematical problems (simple ones, when we can answer the problem's question using only one arithmetical operation, and compound, including also typical ones.) We have experimentally demonstrated the efficiency of this method when forming a full ability to solve the problems among schoolchildren.

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Solving plot mathematical problems as the way to acquire key and subject competencies by schoolchildren

The paper states that plot Mathematical problems are the models of the processes that are happening in the world around. Besides, they are a means of forming the younger schoolchildren's key competencies that is to solve tasks that occur in the everyday and professional life. The goals of the problems' solution are considered, the activity of solving a problem is analysed, four stages in the process of the problem's solution and means of their implementation are singled out.

Key words: mathematical competence, primary school, mathematical problems

Translated by Yulia Vtornikova

Решение сюжетных математических задач как способ формирования у младших школьников предметной и ключевых компетентностей

В статье показано, что сюжетные математические задачи являются моделями процессов, происходящих в окружающем мире, и являются средством формирования у младших школьников ключевых компетентностей, а именно – решения задач, возникающих в повседневной и профессиональной жизни человека. Рассмотрены цели решения задач, проанализирована деятельность по решению задачи, выделены четыре этапы в работе над задачей и средства их реализации.

Ключевые слова: математическая компетентность, начальная школа, математические задачи

Tłumaczenie: Светлана Скворцова (Svetlana Skvortsova)

Rozwiązanie zadań matematycznych z treścią jako metoda formowania u uczniów w wieku wczesnoszkolnym kompetencji przedmiotowych i kluczowych

Artykuł pokazuje, że zadania matematyczne z treścią są modelami procesów,zachodzących w otaczającym świecie i są środkiem do kształtowania u uczniów w wieku wczesnoszkolnym kluczowych kompetencji, w szczególności: rozwiązywania problemów, z którymi ludzie spotykają się w życiu codziennym i zawodowym. W artykule zostały rozpatrzone cele rozwiązania zadań, została przeanalizowana działalność na rzecz rozwiązania zadań, wskazano cztery etapy w pracy nad zadaniem i środki ich realizacji.

Słowa kluczowe: kompetencje matematyczne, edukacja wczesnoszkolna, zadania matematyczne

Tłumaczenie: Eugenia Smyrnova-Trybulska