

USING KERNEL FUNCTION AND A-STABLE DISTRIBUTION FOR DETERMINING VALUE AT RISK (VaR) FOR THE COMPANIES INCLUDED IN WIG20 INDEX AT WARSAW STOCK EXCHANGE

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Abstract

One of the most common from among conducted studies related to capital market includes studies regarding methods intended for suitable description of distribution of rates of return. Familiarity with such distributions (density function) is often necessary condition for usability of other methods, techniques and models describing the respective elements of the capital market such as i.a. risk. One method of risk measurement is evaluation of rate of return variability, exactly based on familiarity of its distributions. From a theoretical point of view, adopting Gauss's axiom is not acceptable, despite its high practicability. This paper presents results of the study related to possible use of α -stable distributions and kernel function for modeling rates of return and using these methods for establishing value at risk (VaR).

Keywords risk, rate of return, value at risk, α -stable distribution, kernel function

I. Introduction

Uncertainty and risk are the elements inseparably related with activity of the investors at capital market. According to K. Jajuga (2007), each investment means giving up current consumption for future and uncertain benefits. A risk results from variability, i.e. uncertain rate of return, which may deviate (up or down) from its expected (the most likely) level (see Sławiński 2006). Depending on the fact, whether risk factors are included in the models of risk measurement or not, a risk may be treated as variability of rate factors or as sensitivity to risk factors. If a risk is perceived as variability (fluctuation) of rate of return, only size of this change in rate of return is measured, without analysis of factors causing these changes. Measurements used for this purpose establish risk value based on distributions of rates of return. Perceiving risk, as sensitivity to factors causing it, degree of variability in rates of return is evaluated as result of action made by a specific factor. Within this scope, specific regression functions are used, which are describing relationship between rate of return and levels of the respective risk factor values (see Jajuga et al. 2008). One of more popular risk measurements, based on variability of rates of return, is VaR (Value-at-Risk). Despite the fact that this measurement is mainly used by financial institutions, it may be used without major problems in investment risk evaluation also by individual investors due to its simplicity and easy interpretation. VaR is the most frequently defined as the highest loss, which should be expected while investing in a given security or investment portfolio, in a given time frame and at adopted level of confidence with simultaneous assumption that the market will behave in "normal" way.

In terms of value, VaR may be defined as:

$$P(W_{t+\tau} \leq W_t - VaR_t) = \alpha \quad (1)$$

where: $W_{t+\tau}$ – value of security (portfolio) at the end of period $t+\tau$ (in practice $\tau=1$ day);
 W_t – current value of security (portfolio);
 α – probability of reaching or exceeding VaR (usually 0.05 or 0.01).

In terms of percentage, VaR is identified as suitable quantile for distribution of rates of return R on investment in a given security (portfolio) and it is expressed as:

$$\text{VAR} = -R_{\alpha} W_t \quad (2)$$

In practice, three main methods of VaR determination, which are the most frequently used, include:

- analytical method (parametric, variance/covariance) – VaR is determined as a specific quantile, assuming that rates of return (and covariance matrix in case of portfolio investment) have normal distribution or log normal distribution (see Dave et al. (1998), Aniūnas et al. (2009)),
- historical simulation method – value at risk is determined based on existing distribution of rates of return for a given security or portfolio (see Hull et al. (1998), Pritsker (2006)),
- Monte Carlo simulation method based on computer simulation (see Hacura et al. (2001), Glasserman et al. (2000)).

Alternative methods for determination of value at risk include i.a. an approach based on determining quantile of any distribution (see Aas et al. (2006), Bednarz (2013)), an approach based on the extreme value theorem (see Danielsson et al. (2000), Embrechts (2000), Bekiros et al. (2005), Bhattacharyya et al. (2008)), an approach based on using value coming from the tail area of a distribution or scenario analysis (see Jamshidian et al. (1996)).

The article presents results of the studies on possible use for estimating value at risk VaR of the kernel function and alpha-stable distributions.

II. Methods

II.1. Kernel function

Kernel estimator for probability density of one-dimensional random variables is defined by the following formula:

$$\hat{f}(x) = \frac{1}{mh} \sum_{i=1}^m K\left(\frac{x-x_i}{h}\right) \quad (3)$$

where: m- random sample size,
h- smoothing parameter,
K- kernel of the function, which meets the following conditions:

$$\int_{-\infty}^{\infty} K(x) dx = 1, \quad (4)$$

$$K(x) = K(-x) \text{ for each } x \in \mathbb{R}, \quad (5)$$

$$K(0) \geq K(x) \text{ for each } x \in \mathbb{R}. \quad (6)$$

Using kernel estimation in order to estimate density function, choice of kernel form and suitable determination of smoothing parameter are very important. From among many known functions, which meet aforementioned requirements for kernel function, final choice is

mainly made based on efficiency of a given function and based on additional properties of a given function. Efficiency of a given kernel means degree of minimization in value of MISE (*Mean Intergrated Square Error*), defined as

$$MISE = \int_{-\infty}^{\infty} E \left([\hat{f}(x) - f(x)]^2 \right) dx. \quad (7)$$

From this point of view, the most effective is the Epanechnikov kernel, defined as:

$$K(x) = \begin{cases} \frac{3}{4}(1-x^2) & \text{for } x \in [-1; 1] \\ 0 & \text{for } x \in (-\infty; -1) \cup (1; \infty) \end{cases} \quad (8)$$

and other, also frequently used kernel, which is normal kernel expressed as

$$K(x) = \frac{1}{\sqrt{2\pi}} \exp \left(-\frac{x^2}{2} \right) \quad (9)$$

is less effective than the Epanechnikov kernel only by 5%. In case of other popular kernel functions (e.g. biweight, triangle), decrease in efficiency in reference to Epanechnikov kernel, is also significantly small. Also for this reason, while selecting kernel function, its properties are mainly considered, such as regularity class, calculation simplicity, or other kernel features, which are crucial for conducted studies (e.g. normal kernel has any derivative, but its integral cannot be expressed with analytical formula, and in case of other kernels, it is possible to calculate an integral, but its derivative does not exist within the whole domain) (Kulczycki (2005)). However, selection of the kernel itself is not that important for quality of estimation, as selection of smoothing parameter h . Adopting value of smoothing factor, which is too high, results in excessive smoothing of estimated probability distribution density function, but adopting value, which is too low, results in occurring high number of local extremes. There are a few methods for determining value of smoothing parameter h . The simplest one is an *approximate method*, which is useful only if estimated distribution is approximate to normal distribution. However, in this case, one should consider its usefulness in conducted studies of kernel density estimation. Other method is *the k-th row substitution method*, which may be used only in case of distribution of one-dimensional random variables, and additionally, if adopted kernel function, which is differentiable $(2k+2)$ -fold. Due to the fact that in practical use of this method it is assumed that $k=2$ (i.e. 2-nd row method is used), it is sufficient that only six first derivatives exist for kernel function. A method as e.g. *cross-validation method*, which may be used regardless of dimension of analyzed random variable, is more universal, but practically it is mainly used in case of multi-dimensional distributions. In addition, regardless of method for determination of smoothing parameter, modification of smoothing parameter is performed for better adjustment of estimated density function to distribution of analyzed variable. It means that in determining value of kernel estimator, kernel determined for each i -th variable is corrected by value of parameter $S_i > 0$, determined based on the formula:

$$S_i = \left(\frac{\hat{f}(x_i)}{\bar{s}} \right)^{-c} \quad (10)$$

where c is non-negative parameter deciding on intensity of modification with the most frequently adopted value of 0.5, $\hat{f}(x_i)$ – it is the kernel estimator established on the base of the formula (3), and \bar{s} is geometric mean of these estimators, obtained as a result of

$$\tilde{s} = \exp\left(\frac{1}{m} \sum_{i=1}^m \ln(\hat{f}(x_i))\right). \quad (11)$$

Considering occurrence of modifying parameter S_i , the kernel estimator for one-dimensional random variable is defined with the formula

$$\hat{f}(x) = \frac{1}{mh} \sum_{i=1}^m \frac{1}{s_i} K\left(\frac{x-x_i}{s_i h}\right). \quad (12)$$

II.2. A-stable distribution

Basic problem with use of α -stable distributions in modeling distribution for rates of return is lack of analytical form describing density of random variable characterized by such distribution. In such case, numerical integration (Fourier transform) of characteristic function is used. If it is necessary to obtain cumulative distribution function of the α -stable distribution, another numerical integration should be performed, which besides complexity of the process, results in accumulation of calculation errors (Purczyński (2003)).

Characteristic function of random variable with α -stable distribution is the following function:

$$\Phi(t) = \begin{cases} \exp\left[i\mu \cdot t - c|t|^\alpha \cdot \left(1 - i\beta \cdot \operatorname{sgn}(t) \cdot t g \cdot \left(\frac{\alpha\pi}{2}\right)\right)\right] & \text{dla } \alpha \neq 1 \\ \exp\left[i\mu \cdot t - c|t| \cdot \left(1 + i\beta \cdot \operatorname{sgn}(t) \cdot \frac{2}{\pi} \ln(t)\right)\right] & \text{dla } \alpha = 1 \end{cases} \quad (13)$$

where: $\alpha \in (0, 2)$ – stability parameter

$\beta \in (-1, 1)$ – skewness parameter

$c > 0$ – scale parameter

μ -shift parameter

$i = \sqrt{-1}$

$\operatorname{sgn}(t) = \frac{t}{|t|}$

Estimation methods for the respective parameters of the characteristic function of the α -stable distribution are presented in publication by Kuruoglu (2001). The log-moment method, which relates to distribution with shift parameter of $\mu=0$, is one of methods proposed in this publication. In order to meet aforementioned assumption, analyzed rates of return should be centered by deducting arithmetic mean from its value. Then, the remaining parameters are obtained from the following formulas:

$$\alpha = \left(\frac{L_2}{\varphi_1} - \frac{1}{2}\right)^{-\frac{1}{2}} \quad (14)$$

$$\beta = \frac{t g(\theta)}{t g\left(\frac{\alpha\pi}{2}\right)} \quad (15)$$

$$c = \cos(\theta) \exp\left((L_1 - \varphi_0)\alpha + 1\right) \quad (16)$$

where: $\varphi_0 = -0.57721566$,

$$\varphi_1 = \frac{\pi^2}{6}$$

$$L_1 = \frac{1}{n} \sum_{k=1}^n \ln(|x_k|)$$

$$L_2 = \frac{1}{n} \cdot \sum_{k=1}^n [\ln(|x_k|) - L_1]^2$$

$$|\theta| = \left(\left(\frac{\varphi_1}{2} - L_2 \right) \alpha^2 - \varphi_1 \right)^{\frac{1}{2}}$$

Using characteristic function of the α -stable distribution, distribution density function of analyzed rates of return is then obtained from the following formula:

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itx} \varphi(t) dt. \quad (17)$$

II.3. Kupiec test

Due to lack of unambiguous criterion of selection of suitable method for estimating value at risk VaR, and a procedure, which is inseparably related to its determination, is verification of used approach. The most common method applied for this purpose is backtesting method, including analysis of estimated VaR exceedance in the past.

Assuming that for each t period, such I_t value may be obtained:

$$I_t = \begin{cases} 0, & \text{when value at risk is not exceeded} \\ 1, & \text{when value at risk is exceeded} \end{cases} \quad (18)$$

then, obtained model well estimates VaR, if obtained number of exceedances is close to assumed level of probability of reaching VaR or its exceedance (comparing with formula 1). If number of exceedances is too high or too low, then the model underestimates or overestimates VaR. The most popular test in this area is the Kupiec test (1995), which null hypothesis assumes that actual number of exceedances is consistent with adopted probability level ($H_0: f = \alpha$), but alternative hypothesis assumes that it is significantly different ($H_1: f \neq \alpha$, where f means share of exceedances in achieved returns). Testing statistics is defined in the following form:

$$LR_K = 2 \left(\ln \left(\left(\frac{T_1}{T} \right)^{T_1} \left(1 - \frac{T_1}{T} \right)^{T-T_1} \right) - \ln(\alpha^{T_1} (1 - \alpha)^{T-T_1}) \right) \quad (19)$$

where T_1 is number of exceedances of VaR with number of observations amounted to T ($T_1 = \sum I_t$).

If null hypothesis is true, LR_K statistics has asymmetric χ^2 distribution with one degree of freedom, which means that at the level of $\alpha=0.05$, critical value amounts to 3.841459.

II.4. Christoffersen test

Kupiec test discussed above only checks, whether number of actual exceedances of value at risk through achieved rates of return is significantly different from adopted probability level. Practically, despite number of exceedances, it is also important that their possible occurrence is random, and independent in time. If exceedances are not evenly distributed in time and they “accumulate” in some periods, it means a significant defect in obtained model.

Within the scope of testing independence in number of exceedances, Christoffersen test using idea of Markov chains is the most common. Null hypothesis assumes that exceedances are independent in time, and alternative hypothesis includes dependence of such exceedances in time, and statistics of Christoffersen test is defined as¹:

¹ The formula has been derivated based on publication [6]

$$LR_{Ch} = 2\ln\left(\left(1 - \frac{T_{01}}{T_0}\right)^{T_{00}} \cdot \left(\frac{T_{01}}{T_0}\right)^{T_{01}} \cdot \left(1 - \frac{T_{11}}{T_1}\right)^{T_{10}} \cdot \left(\frac{T_{11}}{T_1}\right)^{T_{11}}\right) - 2\ln\left(\left(\frac{T_1}{T}\right)^{T_1} \cdot \left(1 - \frac{T_1}{T}\right)^{T_0}\right) \quad (20)$$

where: T_{01} – number of periods, for which $I_t=1$ and $I_{t-1}=0$;

T_{10} – number of periods, for which $I_t=0$ and $I_{t-1}=1$;

T_0 – number of periods, for which VaR was not exceeded ($T_0=T-T_1$),

Similarly to Kupiec test, Christoffersen test has asymptotic χ^2 distribution with one degree of freedom.

III. Data and study results

Subject of the study included daily rates of return for 20 companies included in WIG20 index. As of 1.10.2013, the companies included: ASSECOPOL, BOGDANKA, BRE, BZWBK, EUROCASH, GTC, HANDLOWY, JSW, KERNEL, KGHM, LOTOS, PEKAO, PGE, PGNIG, PKNORLEN, PKOBP, PZU, SYNTHOS, TAURONPE and TPSA. For each company, the study period included time between the second quotation (possible establishing of the first daily rate of return) and 30.09.2013. The studies were conducted based on arithmetic rate of return, which was determined according to the following formula:

$$R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} \cdot 100 \quad (21)$$

where:

P_t -security price within the period t ;

P_{t-1} -security price within the period $t-1$;

D_t - value of paid dividend within the period t .

III.1. Evaluation of distribution normality for daily rates of return

Conducted preliminary analysis using i.a. Kolmogorov-Lilliefors test (K-L) indicates that in case of all analyzed companies, hypothesis on distribution normality for their daily rates of return should be rejected. Except for JSW and PGE, the companies recorded positive mean daily rates of return; in case of all companies, there were leptokurtic distributions, and in case of 15 companies, they were additionally characterized by right asymmetric distribution.

Table 1. Numerical characteristics of daily rates of return of analyzed companies.

#	company	n	mean	standard deviation	kurtosis	skewness	min	max	Lilliefors Test (K- L)
1	ASSECOPOL	3847	0.06041	2.73411	5.86509	-0.0977	-26.33229	15.74468	K- L=0.070278; p<1E-07
2	BOGDANKA	1075	0.07609	1.73988	17.79698	1.59189	-7.30435	20.15722	K- L=0.085511; p<1E-07
3	BRE	5059	0.13831	2.96131	3.22656	0.15250	-14.72868	17.27273	K- L=0.081586; p<1E-07
4	BZWBK	4956	0.12245	2.99950	4.18374	0.08929	-20.47646	20.00000	K- L=0.083166; p<1E-07
5	EUROCASH	2169	0.14710	2.37929	2.68344	0.44369	-9.88372	13.04348	K- L=0.072103; p<1E-07
6	GTC	2364	0.02637	2.78641	3.75017	0.38789	-13.63636	18.86282	K- L=0.067611; p<1E-07
7	HANDLOWY	4075	0.05045	2.19338	5.91279	0.07921	-18.2058	16.66667	K- L=0.086812; p<1E-07
8	JSW	562	-0.07720	2.49255	5.03845	0.23883	-10.44776	16.27907	K- L=0.05944; p=6.59252E-05
9	KERNEL	1467	0.09150	2.88967	4.83163	0.30473	-18.95262	17.41935	K- L=0.081002; p<1E-07
10	KGHM	4069	0.08341	2.94114	4.49956	0.01850	-21.04097	19.67213	K- L=0.053299; p<1E-07
11	LOTOS	2088	0.03792	2.37290	3.83651	0.29024	-10.00000	18.62069	K- L=0.039476; p<1E-07
12	PEKAO	3830	0.06026	2.39692	4.12554	0.23948	-18.60465	16.32124	K- L=0.049126; p<1E-07
13	PGE	979	-0.02758	1.70682	3.53539	-0.32144	-9.21241	8.45070	K- L=0.062626; p<1E-07
14	PGNIG	2013	0.04190	1.95639	1.86318	0.23298	-7.73196	9.62025	K- L=0.059312; p<1E-07
15	PKNORLEN	3474	0.04362	2.15862	1.91823	0.08989	-11.44781	13.72998	K- L=0.035724; p<1E-07
16	PKOBP	2233	0.04276	2.16683	2.28540	0.09322	-11.50519	10.48689	K- L=0.047718; p<1E-07

17	PZU	853	0.03237	1.50840	2.47448	-0.21051	-7.53345	7.27273	K- L=0.050914; p=1.9299E-05
18	SYNTHOS	2206	0.13308	2.64950	4.12986	0.13239	-18.36735	15.87302	K- L=0.119758; p<1E-07
19	TAURONPE	819	0.00812	1.65776	3.66877	-0.41586	-9.72222	7.83410	K- L=0.055386; p=3.1882E-06
20	TPSA	3730	0.00762	2.22403	9.03398	-0.42548	-27.92096	13.63636	K- L=0.054528; p<1E-07

Source: own elaboration.

III.2. Determining value at risk

Also obtained results in determining value at risk indicate false *a priori* assumption on normality of rates of return. Adopting probability level of 0.05 and assuming normality rates of return, Kupiec test indicated incorrectly determined VaR for all evaluated companies. Furthermore, in case of such companies as Bogdanka, GTC or Tauronpe, determined value at risk was lower than recorded highest negative daily rate of return.

Table 2. Determined VaR together with values of Kupiec test and Christoffersen test assuming normal distribution of daily rates of return.

#	company	VaR	LR _k	LR _{ch}
1	ASSECOPOL	-0.1036	308.60237	4.97259
2	BOGDANKA	-0.0917	-	-
3	BRE	-0.0913	223.8565	56.94556
4	BZWBK	-0.1298	458.87941	0.01212
5	EUROCASH	-0.0719	140.09992	0.16887
6	GTC	-0.1730	-	-
7	HANDLOWY	-0.1142	378.18914	0.00983
8	JSW	-0.0625	14.1087	11.22404
9	KERNEL	-0.1494	139.80214	0.00273
10	KGHM	-0.0721	164.09749	11.74177
11	LOTOS	-0.0700	137.1191	0.15032
12	PEKAO	-0.0676	234.58314	1.80987
13	PGE	-0.0661	72.00225	0.04104
14	PGNIG	-0.0683	172.30157	0.01991
15	PKNORLEN	-0.0621	241.70521	0.17706
16	PKOBP	-0.0802	194.04008	0.01795
17	PZU	-0.0701	77.89889	0.0047
18	SYNTHOS	-0.0995	191.36765	0.01817
19	TAURONPE	-0.1568	-	-
20	TPSA	-0.2123	370.0888	0.00107

Source: own elaboration.

Slightly better results were obtained in case of adopting assumption about α -stable distribution of rates of return. In 11 out of 20 cases, Kupiec test indicated no basis for rejection of null hypothesis, in other cases, overestimation of value at risk took place. In case of Christoffersen test, only in case of 5 companies, there was no basis for rejection of H_0 . It mainly results from the fact that analyzed companies frequently revealed sequence of exceedances of determined VaR (T11). In case of BRE bank, 35 of such moments were recorded, when in at least two consecutive quotations, the company's security price decreased more than determined value at risk. In case of BZWBK bank, there were 33 of such cases and in case of ASSECOPOLand and HANDLOWY bank – 29 (table 3). BRE bank is also a company, which produced the longest series consecutive determined value at risk. In five consecutive quotations, within a period between 1993-06-01 and 1993-06-09, the company's security price decreased from PLN 10.20 to PLN 6.72 (by 34.12%), but each time, decrease was not smaller than determined VaR. Similar situation also occurred in case of HANDLOWY bank, which within five consecutive quotations, between 1998-08-26 and 1998-09-01, lost 26.92% in total (price decreased from PLN 53.50 to PLN 39.10).

Table 3. Determined VaR together with values of Kupiec test and Christoffersen test assuming α -stable distribution of daily rates of return.

#	company	VaR	LR _k	LR _{ch}	T11
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1	ASSECOPOL	-0.0444	4.96042	45.42637	29
2	BOGDANKA	-0.0279	1.23406	-0.00095	2
3	BRE	-0.0473	13.02833	58.45028	35
4	BZWBK	-0.0481	8.24698	53.80803	33
5	EUROCASH	-0.0377	3.49839	5.97236	9
6	GTC	-0.0456	0.77305	23.1611	18
7	HANDLOWY	-0.0356	6.22098	43.87025	29
8	JSW	-0.0418	2.06162	3.8889	3
9	KERNEL	-0.0466	2.71972	0.84545	4
10	KGHM	-0.0475	10.01135	20.83802	20
11	LOTOS	-0.0387	1.12667	32.42173	19
12	PEKAO	-0.0388	10.71394	21.06703	19
13	PGE	-0.0284	2.78421	3.14682	4
14	PGNIG	-0.0318	0.00128	0.00248	5
15	PKNORLEN	-0.0351	5.27604	14.85859	17
16	PKOBP	-0.0352	4.27964	15.88959	13
17	PZU	-0.0245	3.68621	2.23982	3
18	SYNTHOS	-0.0422	1.04379	11.84815	13
19	TAURONPE	-0.0272	0.02811	5.17597	6
20	TPSA	-0.0365	6.72976	7.75004	14

Source: own elaboration.

Undoubtedly, the best results in determining value at risk were obtained using kernel function for this purpose, not only without modification of smoothing parameter, but also with its modification. In both cases, Kupiec test indicated that determined VaR for the respective companies does not significantly differ from adopted probability level (there was no case, which provided basis for rejection of null hypothesis). It was obtained mainly due to determining value at risk at slightly lower level, than it was in case of α -stable distribution. However, it resulted in lower number of companies, for which there was no basis for rejection of null hypothesis in case of Christoffersen test from five to four [compare with table 4 and table 5].

Table 4. Determined VaR together with values of Kupiec test and Christoffersen test using kernel function without modification of smoothing parameter.

#	company	VaR	LR _k	LR _{ch}	T11
1	ASSECOPOL	-0.0413	0.15803	37.73766	31
2	BOGDANKA	-0.0264	0.09786	0.39305	2
3	BRE	-0.0430	0.41721	52.04235	41
4	BZWBK	-0.0442	0.03342	65.4466	44
5	EUROCASH	-0.0354	0.29426	7.07735	13
6	GTC	-0.0444	0.04336	19.50845	18
7	HANDLOWY	-0.0327	0.17235	49.88467	36
8	JSW	-0.0397	0.37328	8.25551	5
9	KERNEL	-0.0437	0.00176	0.4739	5
10	KGHM	-0.0444	0.29055	20.12572	25
11	LOTOS	-0.0382	0.56494	30.2481	19
12	PEKAO	-0.036	0.00138	22.23166	26
13	PGE	-0.0271	5E-05	6.3147	7
14	PGNIG	-0.0319	0.00128	0.00248	5
15	PKNORLEN	-0.0337	1.16692	14.42621	19
16	PKOBP	-0.0336	0.90301	11.31601	13
17	PZU	-0.0232	0.55311	0.83967	3
18	SYNTHOS	-0.0405	0.00086	12.78338	15
19	TAURONPE	-0.0268	0.10636	4.77032	6
20	TPSA	-0.0344	0.06955	6.33342	17

Source: own elaboration.

Table 5. Determined VaR together with values of Kupiec test and Christoffersen test using kernel function with modification of smoothing parameter.

#	company	VaR	LR _k	LR _{ch}	T11
1	ASSECOPOL	-0.0409	0.01	35.58825	31
2	BOGDANKA	-0.0258	0.34524	0.55854	2
3	BRE	-0.0427	0.20278	53.39444	42
4	BZWBK	-0.0441	0.03342	65.4466	44

5	EUROCASH	-0.0347	0.54152	6.52512	13
6	GTC	-0.0439	0.00569	18.06874	18
7	HANDLOWY	-0.0326	0.07308	48.63748	36
8	JSW	-0.0393	0.0459	6.91038	5
9	KERNEL	-0.0431	0.0388	0.32771	5
10	KGHM	-0.0438	0.0109	18.0196	25
11	LOTOS	-0.0375	0.02569	27.52893	20
12	PEKAO	-0.0354	0.22979	24.17928	28
13	PGE	-0.0267	5E-05	6.3147	7
14	PGNIG	-0.0314	0.05734	0.02145	5
15	PKNORLEN	-0.0331	0.0444	13.03143	20
16	PKOBP	-0.0332	0.06671	11.07955	14
17	PZU	-0.0228	0.3381	0.70047	3
18	SYNTHOS	-0.0404	0.00086	12.78338	15
19	TAURONPE	-0.0264	0.2337	4.38507	6
20	TPSA	-0.0340	0.01267	6.79667	18

Source: own elaboration.

IV. Summary

Study results presented above unambiguously indicate that adopting *a priori* assumption on normal distribution of rates of return at capital market is not recommended action. However, such assumption is often adopted in various types of models, which describe functioning of capital market, or which are used to support investment decisions. It occur in Black Scholes model of option pricing (1973), in capital asset pricing model CAMP (see Shapre (1964)), or in case of some methods for determining value at risk (VaR). Assumption on normal distribution of rates of return is mainly adopted in order to accelerate, simplify and make easier performing particular calculations. However, significant deviation of actual rates of return from the ones adopted in assumptions may result in many negative consequences. It may be i.a. the base for questioning reliability, hence applicability of many techniques, methods and models used for analyses, diagnoses and forecasts of the capital market. For this reason, there is continuous process of searching for new tools and testing tools previously used in other areas, tools describing phenomena, which take place at the capital market, including the ones used in modeling rates of return. However, α -stable distribution adopted for the capital market by Mandelbrot (1963) provides only slightly better results within this area than normal distribution. Based on conducted studies, it may be stated there is high potential related to use of kernel estimation in modeling rates of return and use of such modeling for determining value at risk.

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