MODELLING AND NUMERICAL AND EXPERIMENTAL INVESTIGATIONS OF CONTACT PHENOMENA AND WEAR PROCESSES IN A MECHANICAL FRICTION CLUTCH

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Summary

This work presents numerical and experimental investigations of contact phenomena and wear processes occurring on the contact surface of linings of a mechanical friction clutch. These problems have already been studied earlier, but only simplified mathematical models have been used and applied. Our work takes into account elasticity and abrasive wear properties of material of friction linings rubbing themselves. A general non-linear differential model of wear and a wear model in the integral form are considered. Besides, we present results describing thermal phenomena (heat generation and its propagation) occurring in the considered system. Many interesting results are obtained, illustrated and discussed. Finally, numerical results are also compared with the experimental data.

Keywords: clutch, friction, wear processes, heat generation.

1. Introduction

A clutch is an element of mechanical system used for coupling shafts and transmitting torque between them. In the early period of development of transport, mechanical industry and machine engineering, belts and transmissions were used for transmitting torque between shafts functioning together. However, a need for individual power transmissions and a compact coupling of shafts which function together in a machinery soon arose. Historically, the oldest simple clutches were used for direct connection of coaxial shafts. Present-day demands as well as future trends in technology of producing clutches make many demands with regard to their structure, functioning, strength or life. The basic issues in designing power transmission systems are, among others, increase of productivity and improvement of function of driven machines, as well as enhancing the degree of reliability

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and obtaining better techno-economic indices of these systems. In order to meet these assumptions in mechanical friction clutches, appropriate knowledge of mathematical description of this clutch, as well as phenomena and tribological processes (friction, wear, heat generation and propagation) occurring therein, is important. Such an approach allows us to better project the behaviour of actual systems of this type.

Issues related to movement dynamics and contact phenomena as well as accompanying tribological processes in different types of mechanical systems have been the object of interests and investigations of many scientists for many years. The afore mentioned issues have been considered in different frictional connections, like bars, transmissions, gears, guides, bearings, clutches, brakes and other frictional connections. Different mathematical models have been used in investigations related to dynamics and tribological processes occurring in systems with a friction clutch to describe them. During analysis of system movement dynamics the contact phenomena and accompanying tribological processes were not taken into account. At the analysis stage of these phenomena and processes, inertia of contacting bodies was not taken into consideration. In most cases simplified mathematical models were used therein, as a rule separately for particular issues and without mutual interrelations among them. In the work presented, a friction clutch is treated as the frictional connection of elastic bodies with taking into account elasticity in axial direction of the material of friction linings. A general non-linear differential model of wear and an integral model of wear taking into consideration hereditary and memory processes were used for numerical simulations. Non-uniform contact pressure distributions on the contact surface of clutch linings have been determined for any instant of time. For any instant of time non-uniform distributions of wear of contacting bodies have been calculated. Changes of the moment of friction force transmitted by the clutch resulting from the change of contact pressure distribution have been calculated. A mathematical model allowing determination of non-uniform temperature distribution on the contact surface of friction linings for any instant of time has been considered. The above presented attempt at joining the mentioned issues into one complex tribological system is not an easy task although the results obtained this way should allow a better projection of the behaviour of actual systems of this type.

2. Review and analysis of literature

Dry friction occurring during sliding of one rubbing surface on another one is a complex phenomenon and in general depends on many parameters [16]. The friction phenomenon is accompanied, among others, by formation of stresses, wear of rubbing materials and heat generation. Abrasive wear is a dynamical process [17], which is related to a change of surfaces of bodies moving relative to one another, as a result of mechanical interaction between them. This process depends on many factors and parameters, like geometry of contacting surfaces, normal force applied, rubbing speed, material hardness, etc. [3]. Studies on wear process and its modelling have been carried out for many years [4]. One of the first scientists investigating wear processes was Archard, who proposed a linear model of wear for metals [2]. Since then many mathematical models describing wear processes in friction joints of different type, in different external conditions and for different materials, have come into being yet. More then 300 different models of wear may be found, from simple empirical equations to complex mathematical relationships in literature related to tribology. Numerical calculations of wear processes in friction joints of different type may be found in the works [3], [7], [15], [16], [19] and many others.

In the general case of modelling wear processes in different type of friction connections of mechanical systems general models of abrasive wear are used. According to Archard [2], the model of wear written in the differential form takes the following form

$$\frac{dw(t)}{dt} = K^{(w)} \left| V_r(t) \right| P(t)\,, \tag{1}$$

where t is time, $w(t)$ is wear, $K^{(w)}$ is material wear coefficient, $V_r(T)$ is relative rubbing speed of rubbing surfaces, while $P(t)$ is contact pressure between them. It is the linear model of wear, considering contact pressure and rubbing speed of rubbing surfaces. This model was also used in another type of friction connection, in the work [12]. In this work for modelling processes of abrasive wear of clutch friction linings the general non-linear differential model of wear has been used, described with the equation [16]

$$\frac{dw(t)}{dt} = K^{(w)}(T'(t)) \left| V_r(t) \right|^{\beta} P^{\alpha}(t)\,, \tag{2}$$

where wear coefficient $K^{(w)}(T'(t))$ is a function of temperature $T'(t)$ on the contact surface, and α and β coefficients are quantities dependent on the model of wear, grade of machining and lubrication of rubbing surfaces. Thus, it is the non-linear model of wear, wherein the speed of wear is the non-linear (exponential) function of contact pressure and rubbing speed of rubbing surfaces. The non-linear model of wear presented was used earlier in the work [16] and others.

At variable conditions of external load so called delay effects may be observed [16], [19]. For some frictional materials, in spite of stable conditions of the wear process, the wear coefficient changes with time, e.g. as a result of ageing or wearing-in of friction linings. Then, there is the necessity to use models of wear other than these presented above. An adequate mathematical description of such wear process is the integral model of wear in the form of [7], [16]

$$w(t) = \int_0^t K^{(w)}(T'(t')) \left| V_r(t') \right| K'(t,t') P(t')\, dt'\,, \tag{3}$$

where $K'(t,t') = K'_1(t')K'_2(t-t')$, wherein $K'_1(t') = 1 + c\exp(-\gamma' t')$ and $K'_2(t-t') = 1 - \exp(-\gamma''(t-t'))$ are so called hereditary and memory kernels. The exponential functions in the presented model of wear are responsible for decreasing the speed of wear process, even in stationary conditions. The model of wear (3) for $K'_1(t') = 1$ and $K'_2(t-t') = 1 - \exp(-\gamma''(t-t'))$ was used in the work [13] and in the work [19], where for a model of contact of a thermoelastic layer with a thermally insulated plate the solution was obtained with taking into account wear and heat generation. An integral model of abrasive wear was also applied, among others, in the works [15] and [18].

The friction phenomenon occurring in a friction clutch is also accompanied, apart from wear processes, by processes of heat generation and propagation. In order to describe the processes of heat generation and propagation in the friction clutch more accurately, general relationships describing these processes should be used. One of the basic equations describing thermal phenomena is the heat equation. The heat equation was used for determining temperature in the contact of two thermoelastic layers in the work [14], whereas application of this equation for description of thermal phenomena occurring in a multi-disc friction clutch of C/C composite may be found in the work [20]. In general, heat flux density $q(t)$ generated during effecting of the friction force on the parting surface of materials has the form [1]

$$q(t) = (1-\chi)\mu|V_r(t)|P(t) \ , \tag{4}$$

where χ denotes a part of work done by the friction force which is not converted to heat (this part of work goes e.g. for wearing), μ is friction coefficient, $V_r(t)$ is rubbing speed and $P(t)$ is contact pressure on the contact surface of the bodies. The formula (4) for heat flux density was used e.g. in the work [5], in which an analysis of thermal phenomena in a friction clutch with a ceramic disc was carried out with use of finite element method. The relationship for heat flux density presented was also applied, among others, by the authors of the works [14] and [15]. And for description of heat conduction in a given material, the Fourier law is used

$$q(t) = -k^{(p)} \operatorname{grad} T' \ , \tag{5}$$

where $k^{(p)}$ is thermal conductivity of material and $\operatorname{grad} T'$ is gradient of temperature T'. In turn, flux density of heat exchanged at the boundary between the body and its environment has the form

$$q(t) = \lambda(T' - T'_{ot}) \ , \tag{6}$$

where λ is heat transfer coefficient between the body and its environment, T' is body temperature at the boundary of contact with the environment, while T'_{ot} is ambient temperature. Exhaustive analyses of scientific works related to different kind of heat may be found in the review works [8], [9] and [10]. In the review work [6] there are discussed papers which have been published in recent years, which were devoted to analyses and computations of heat phenomena in friction brakes of different kind, and conclusions from this work, obtained for brakes, may be also compared to friction clutches considering similar structure of these devices.

3. Model of the considered friction clutch

Figure 1 presents a model of two-disc mechanical friction clutch and a cross-section of friction linings of this clutch with a computational grid (plotted on the cross-section of the linings divided into m equal segments along the radius, in nodes of which temperature values are being determined).

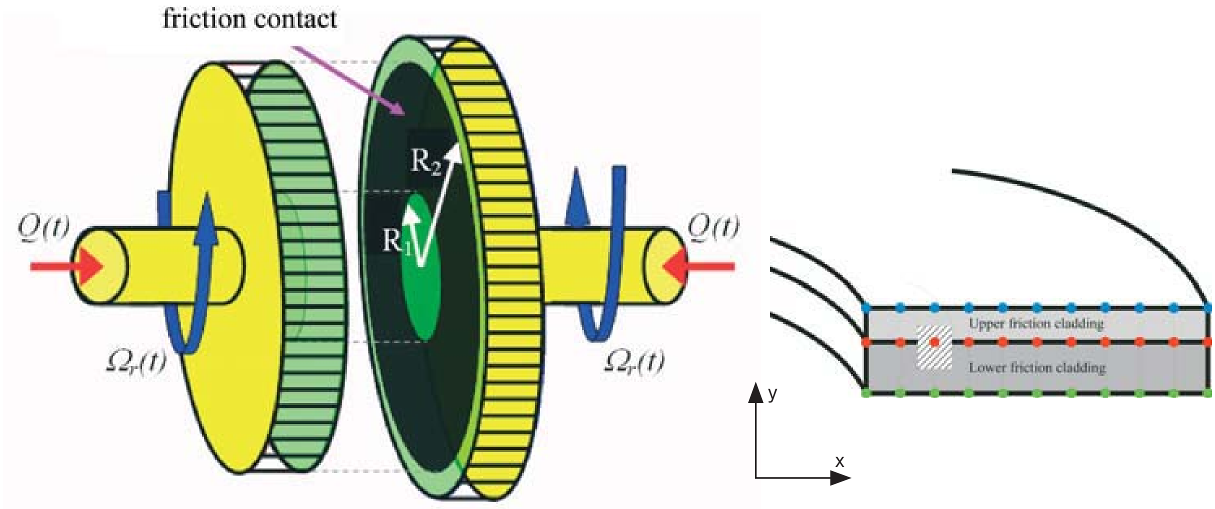

Fig. 1. Model of two-disc mechanical friction clutch and a cross-section of linings of the considered clutch with a plotted computational grid.

The friction linings are attached to both discs of the clutch. The friction contact between these linings occurs in the ring area $R \in [R_1, R_2]$. The discs are being pressed by axial force $Q(t)$ and their relative angular velocity is Ω_r, while contact pressure in any contact point and time is equal to $P(R,t)$. Material wear coefficients for the left and right friction linings, dependent on temperature $T'(R,t)$ in a given contact point and time, are $K_1^{(w)}(T'(R,t))$ and $K_2^{(w)}(T'(R,t))$, respectively. Stiffness coefficients, in turn, in axial direction of material of these linings are k_1 and k_2, respectively. Thicknesses of the upper and lower linings are H_1 and H_2, respectively. Thermal conductivities of respective linings are $k_1^{(p)}$ and $k_2^{(p)}$. Heat transfer coefficients between the upper (lower) friction lining and respectively the upper (lower) clutch disc made of aluminium are λ_1 and λ_2, respectively. Heat transfer coefficients between the upper/lower lining and environment in turn are λ_3 and λ_4, respectively. Specific heats of materials of which the linings are made are c_{w1} and c_{w2}, respectively, while densities of material of which these linings are made are ρ_1 and ρ_2. The detailed mathematical description of processes of wear and heat generation and propagation in the clutch is presented in the work [1]. Integrals occurring in differential, integral and integro-differential equations obtained in dimensionless form were written with the trapezium method. Appropriate equations were solved using the fourth order Runge-Kutta method and the Gauss-Jordan elimination method. In this work the results of numerical simulations of obtained relationships are presented and compared to the own experimental investigations.

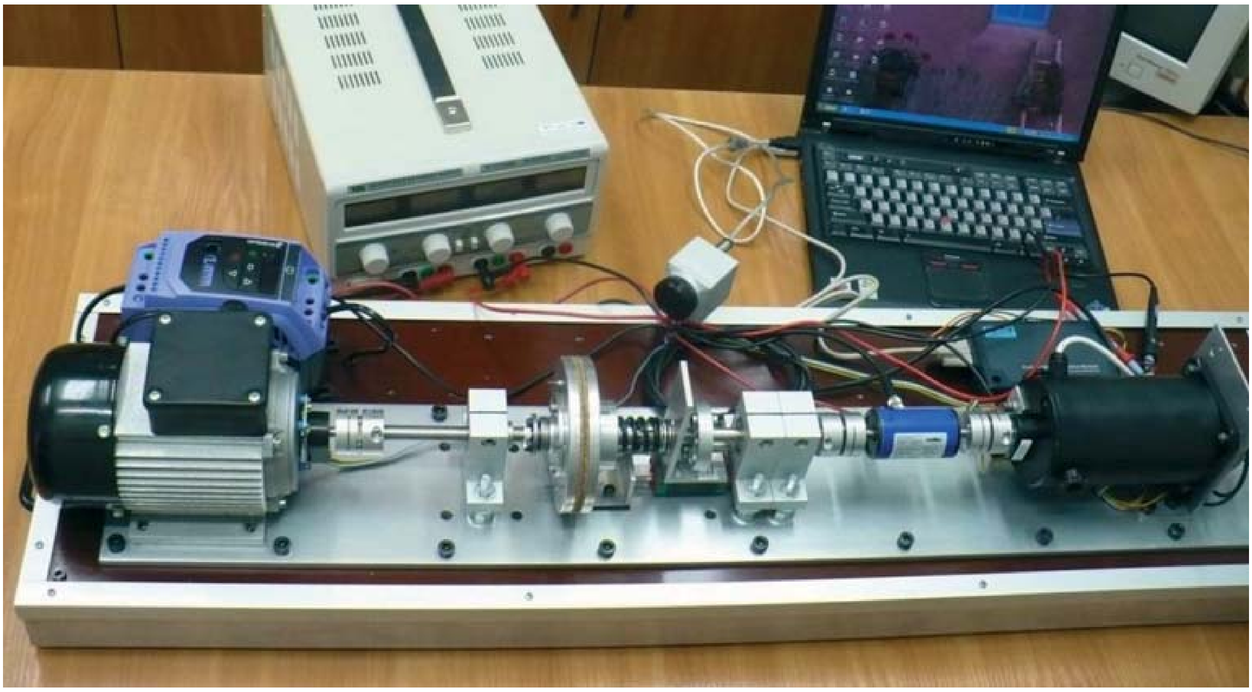

Fig. 2. A general view of the experimental stand.

4. Experimental stand

A general view of the experimental stand is presented in Figure 2.

The stand comprises a mechanical system with a friction clutch operating in a mechatronic system. This is a typical system in which clutches work. It consists of a driving part, driven part and friction clutch. The driving part includes an asynchronous motor controlled with a single-phase AC inverter. For determining angular position of the active part of the clutch (motor) serves as an optical incremental encoder. The driven part includes a DC motor working as a generator which causes appropriate anti-torque depending on a connected load. Furthermore, in the driven part of the system there is a friction brake and an optical incremental encoder. The transmitted torque is measured with a dynamic torque sensor. A member coupling both devices is the mechanical friction clutch. A member coupling the mechanical system with computer software is the control and measuring module USB-4711A.

5. Results of experimental investigations

The results of conducted experimental investigations have been compared to the analytic solutions and numerical computations as well. The friction linings used for investigations were made of pressed cork which is a natural material used for friction linings. In Figure 3 a comparison of the results of numerical simulations with the experimental results is presented, that is: examples of the changes of distance between the discs with time, the

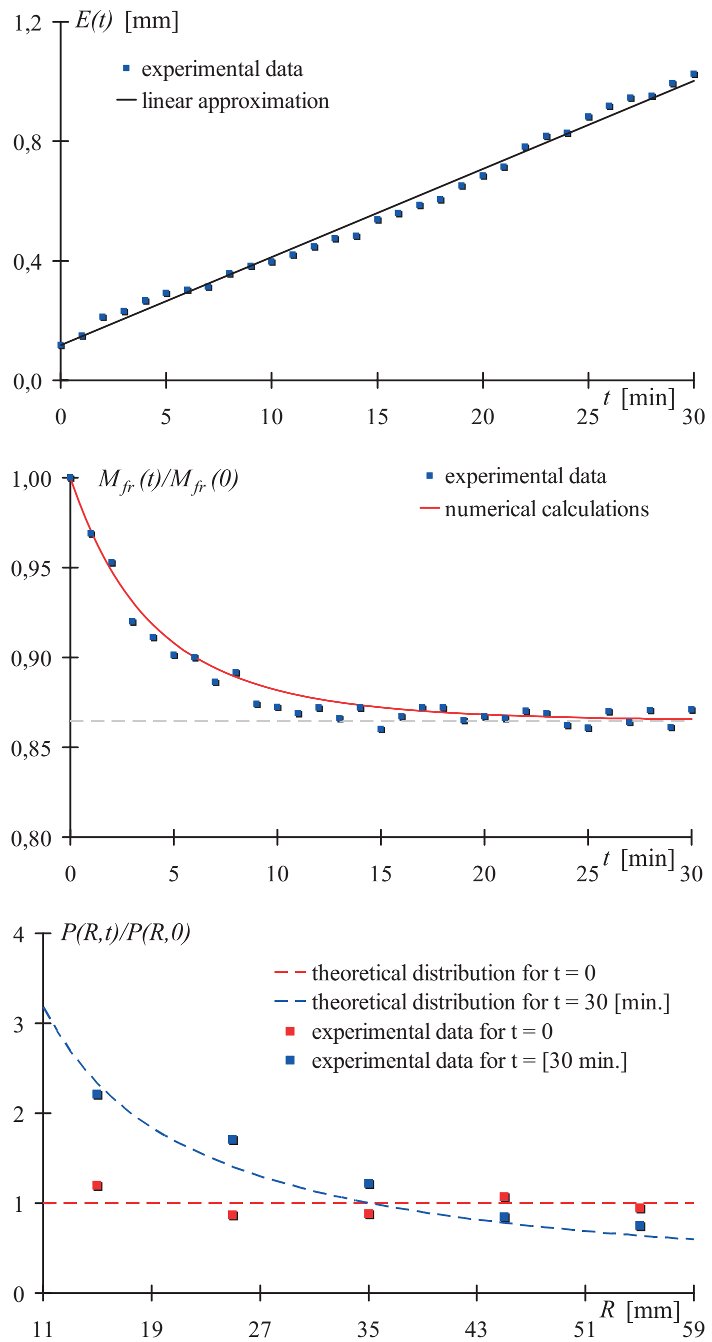

Fig. 3. Comparison of numerical with experimental results: a) the changes of distance between the discs with time; b) the changes of moment of friction force transmitted by the clutch; c) the contact pressure distributions on the contact surface of the linings.

changes of moment of friction force transmitted by the clutch and the contact pressure distributions on the contact surface of the linings.

On the grounds of the results obtained it may be roughly assumed that the wear of the clutch friction linings is proportional to time. According to the numerical solution, the actual values of moment of friction force transmitted by the clutch decrease as the process of wear of the clutch friction linings advances. Even though the results of contact pressure distribution do not correspond exactly with the numerically obtained results, substantial changes of the contact pressure distribution according to the simulations may still be observed.

Figure 4 presents, as examples, the surface distributions and the profiles of temperature on the surface of a new, not used friction lining (for which uniform contact pressure distribution may be assumed) heated to higher and higher temperature. The temperature distributions on the contact surface of the lining are not uniform. For each of the presented cases temperature presented is lower on the inner part of the lining (yellow colour), while higher is at the outer boundary of the lining (red colour). In the presented profiles of temperature it changes along the radius of the clutch friction lining. Moreover, it may be assumed that it changes more or less linearly along the radius.

Figure 5 presents, as examples, the surface distributions and the profiles of temperature along the radius of the lining for the uniform (on the left) and the steady non-uniform (on the

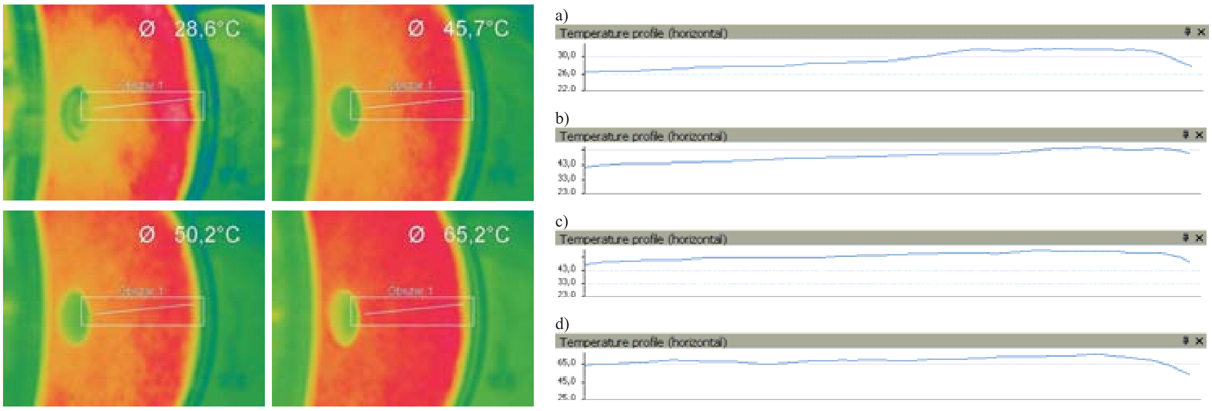

Fig. 4. Surface distributions and profiles of temperature on the contact surface of the friction lining.

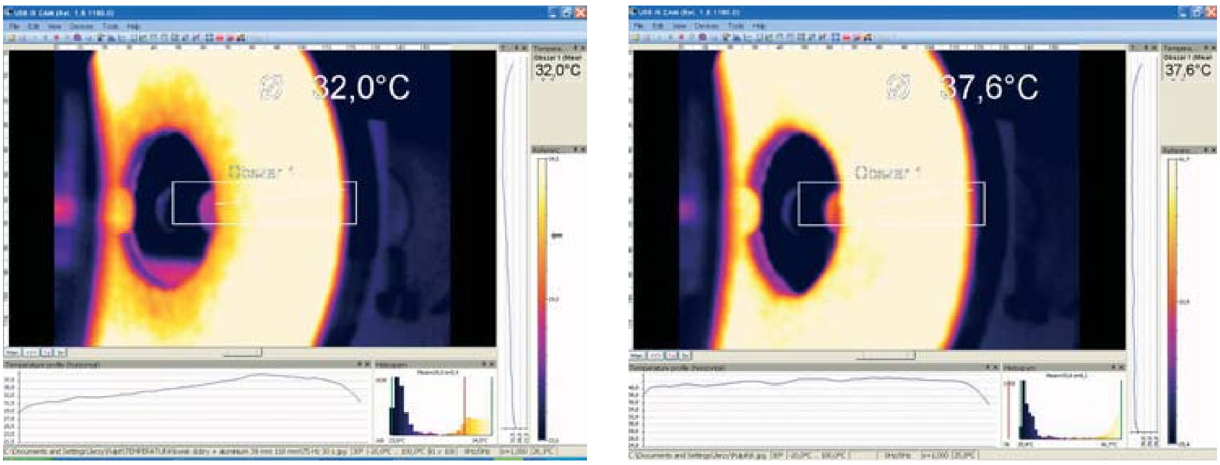

Fig. 5. Surface distributions and profiles of temperature on the contact surface of the friction linings for the uniform (on the left) and the steady non-uniform (on the right) pressure distribution.

right) pressure distribution. Also in this case it is seen clearly that temperature distribution on the contact surface of the friction lining is not uniform. Approximately, temperature increases linearly along the radius of this lining.

Deviations from the linear relationship result from the fact that a heat exchange between the lining and its environment there takes place at the contact boundaries and hence, the values of temperature in these places of the lining are adequately lower. In the second case it is seen that the temperature distribution is approximately uniform on all lining surface. In the actual system the values of temperature are a little lower both at the inner and the outer boundaries which is caused by a heat exchange between the lining and its environment.

6. Results of numerical simulations

For the numerical analysis of wear processes of the clutch friction linings by use of both the differential and the integral models of wear constant wear coefficients and constant friction coefficient were assumed. A symmetrical system consisting of two identical clutch friction linings was considered. Numerical calculations for the differential wear model have been carried out for $\alpha = 1$ and $\beta = 1$. Figure 6 presents, as examples, the distributions of dimensionless contact pressure $p(r,\infty)$ for dimensionless radius r in a steady state for different values of dimensionless geometrical parameter ρ which characterizes the shape of the ring-shaped contact surface of the clutch linings. Moreover, in this figure the homogeneous contact pressure distribution for the initial instant of time is shown.

At the initial instant of time the contact pressures on all contact surface of the linings are the same. As time goes on and the processes of wear of the linings advance, the contact pressure distributions change. In the steady state (theoretically, after an infinitely long time) these distributions take a specific form dependent on geometrical parameter ρ. For large values of ρ parameter, the contact pressure distribution does not change

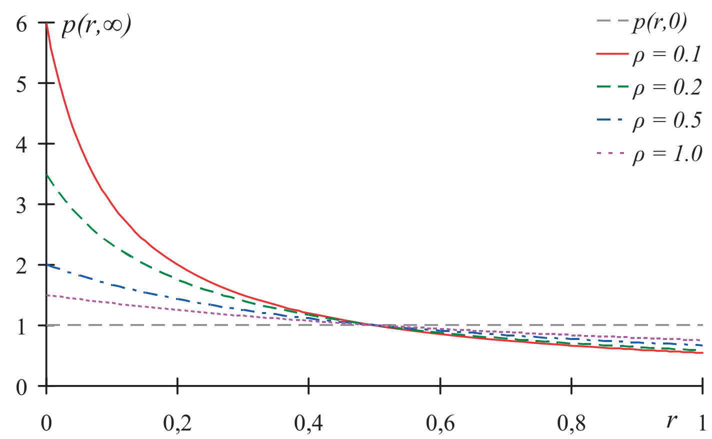

Fig. 6. Contact pressure distributions in the steady state.

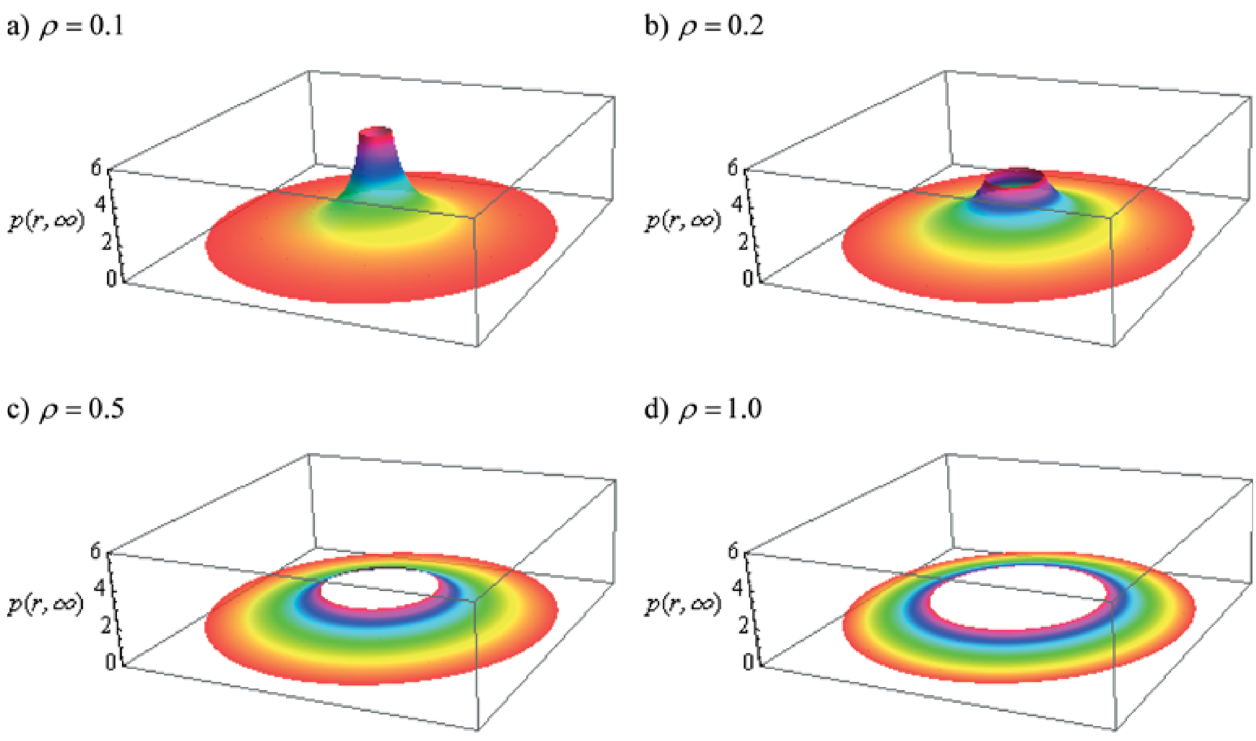

Fig. 7. Visualizations of the contact pressure distributions in the steady state in polar co-ordinate system.

significantly in relation to the initial distribution, whereas differences are considerable for lower values of this parameter. The determined contact pressure distributions are also presented in Fig. 7 as visualizations in polar co-ordinate system, where the domain of a function are surfaces corresponding to the contact surfaces of the friction linings for respective values of geometrical parameter ρ.

Time evolutions of the contact pressure distributions as functions of dimensionless radius r and dimensionless time τ are presented in Fig. 8. For both cases the contact pressure distributions at the initial instant of time are uniform on all contact surface, and they change as wear process advances. Thereafter they achieve a steady distribution dependent on ρ parameter.

In Figure 9 the time evolutions of the total wear distributions $u(r,\tau)$ of these linings are presented. At the initial instant of time the wears in each point of the contact of the linings are zero. As the wear process advances, the wears in individual points of contact of the linings increase.

The changes of the moment of friction force transmitted by the clutch are presented in Fig. 10.

As time goes on the moment of friction force transmitted by the clutch decreases. It results from the change of the contact pressure distribution on the surface of the contacting clutch friction linings. In time of a steady state, when some distribution of the contact

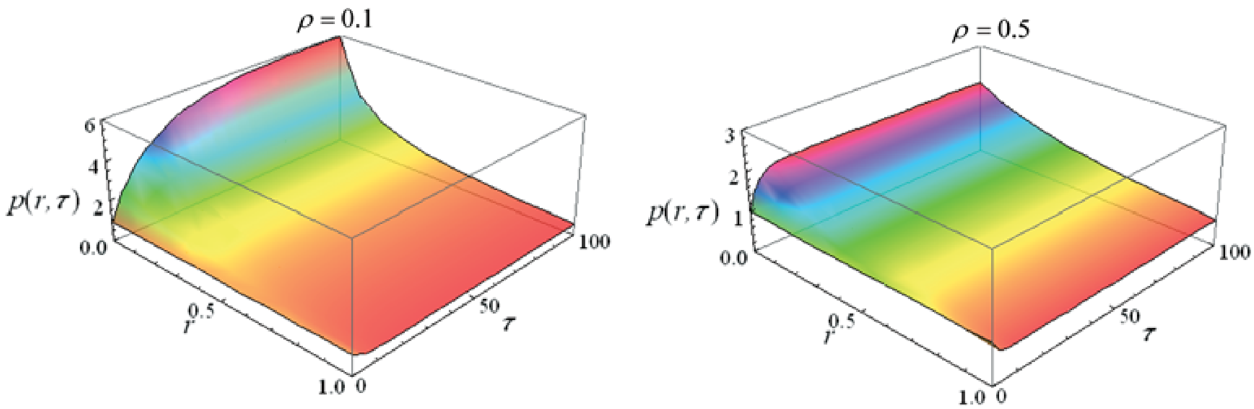

Fig. 8. Time evolutions of the contact pressure distributions.

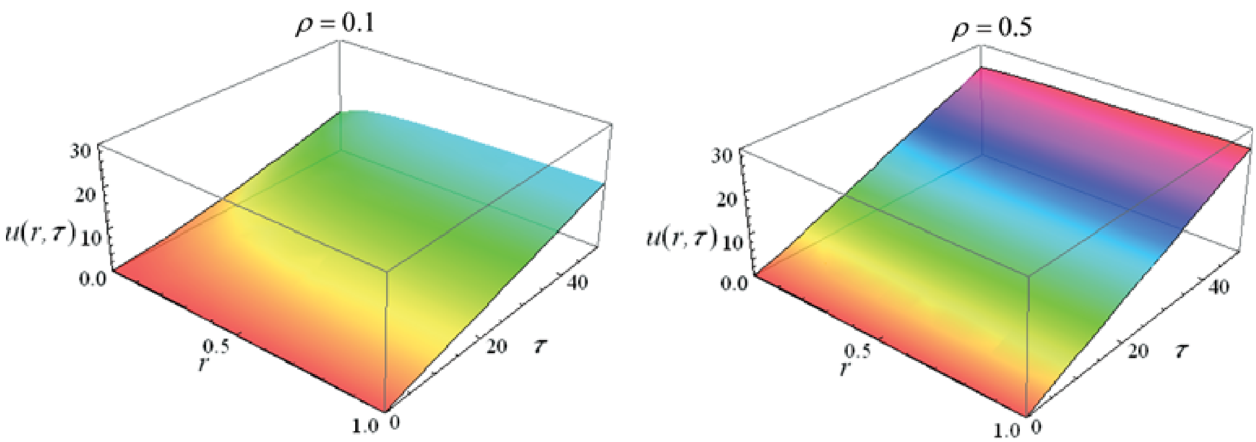

Fig. 9. Time evolutions of the total wear distributions of the linings.

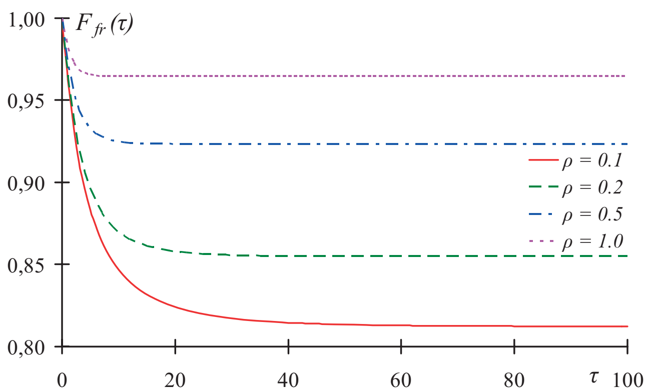

Fig. 10. Changes of the moment of friction force transmitted by the clutch.

pressure is stabilized, the moment of friction force transmitted by the clutch also achieves a steady, stabilized value. The relative change of the value of this moment is larger for smaller values of geometrical parameter ρ.

The numerical calculations were carried out again for the integral model of wear assuming a dimensionless function $K(\tau,\xi)$ with the form $K(\tau,\xi)=\exp(-\gamma(\tau-\xi))$. Figure 11 presents, as examples, the contact pressure distributions in a steady state for different values of γ parameter. These distributions are also presented in Fig. 12 as visualizations in polar co-ordinate system.

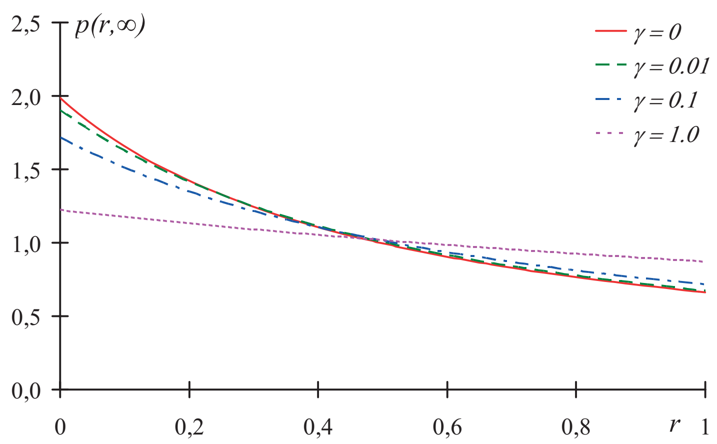

Fig. 11. Contact pressure distributions in a steady state.

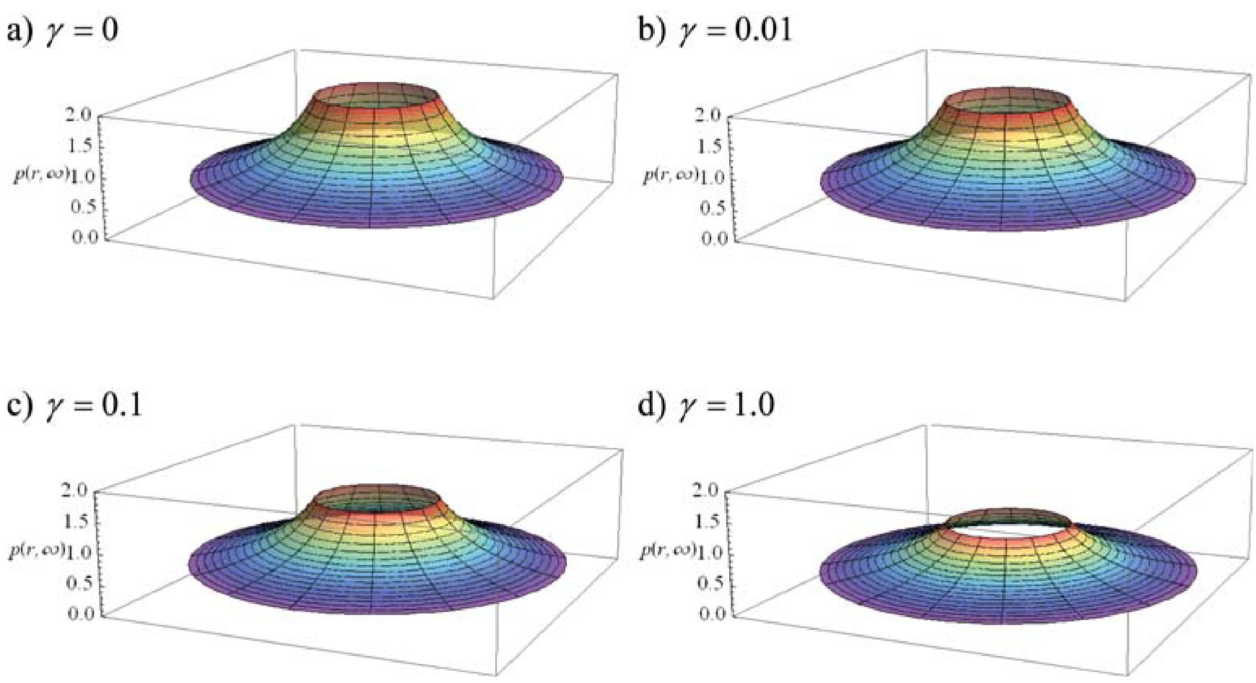

Fig. 12. Visualization of the contact pressure distributions in a steady state.

In the initial instant of time, before the process of wear of the linings starts, the contact pressure distribution is uniform on all contact surface of the linings. However, in the steady state, theoretically after an infinitely long time, the contact pressure distributions for different values of γ parameter vary. For larger values of γ parameter the contact pressure distribution differs less and less from the initial distribution. It corresponds to the case of quicker wearing of the discs for a large value of γ parameter, and consequently the speed of wear quickly decreases and the contact pressure distribution stops to change.

In Figure 13 the changes of the contact pressure distributions, and in Figure 14 the changes of the total wear distributions of the linings for different values of γ parameter are presented.

From the presented figures it can be seen that at the initial instant of time, before the process of wear of the clutch linings starts, the contact pressures are distributed uniformly on the contact surface, and the wear of the linings is zero. As time goes on and the process of wear advances, both the contact pressure distributions and the distributions of wear of the linings change, and this is in different way depending on γ parameter.

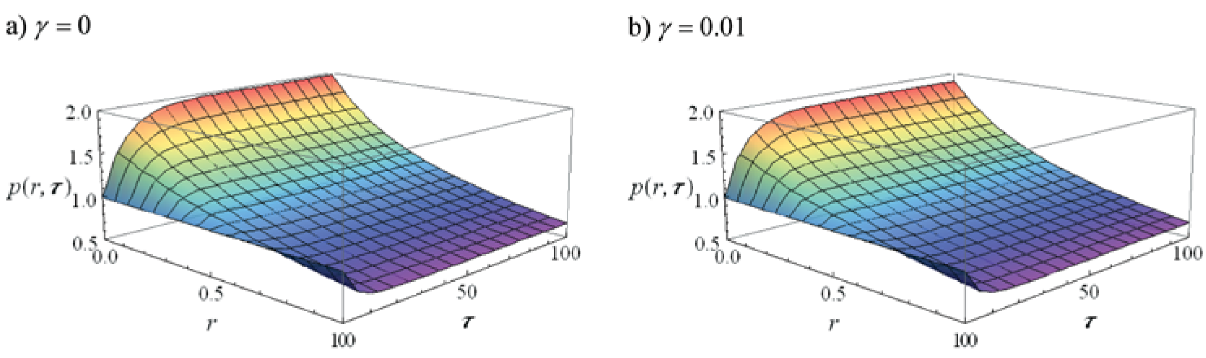

Fig. 13. Time evolutions of the contact pressure.

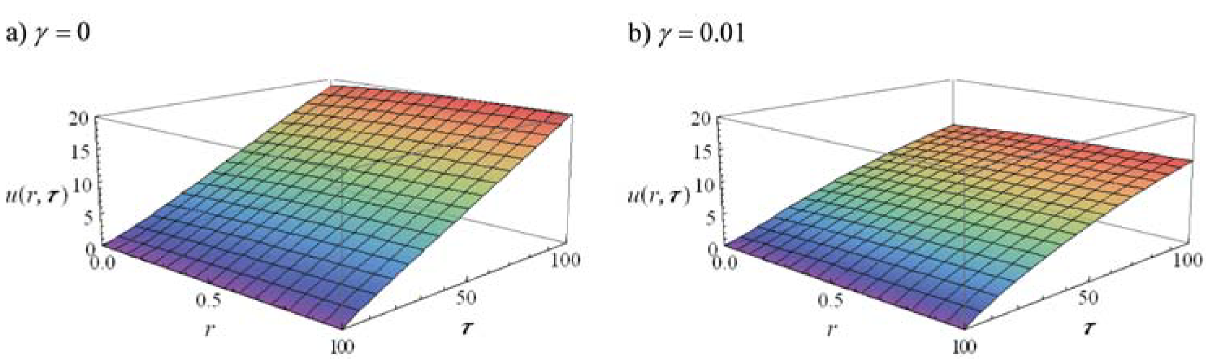

Fig. 14. Time evolutions of the total wear of the linings.

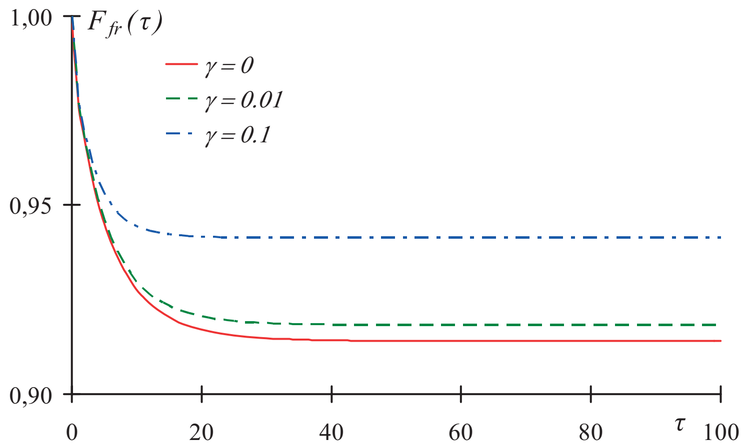

Fig. 15. Changes of the moment of friction force transmitted by the clutch.

In Figure 15 the changes of the moment of friction force transmitted by the clutch are presented yet for different values of γ parameter, as a function of dimensionless time τ. As time goes on the moment of friction force transmitted by the clutch decreases. In the steady state the moment of friction force transmitted by the clutch takes the steady values depending on the γ parameter.

As examples, the numerical results of the model describing thermal phenomena in the friction clutch are presented below. At the beginning the case of pressures uniformly distributed on all contact surface of the clutch friction linings is considered. Figure 16 presents the distributions of dimensionless temperature $T(r,\infty)$ in a steady state, i.e. when these distributions have a steady form on all contact surface of the linings, independently of time. The calculations have been carried out for different values of parameters $c_1 = c_2$ characterizing considered system.

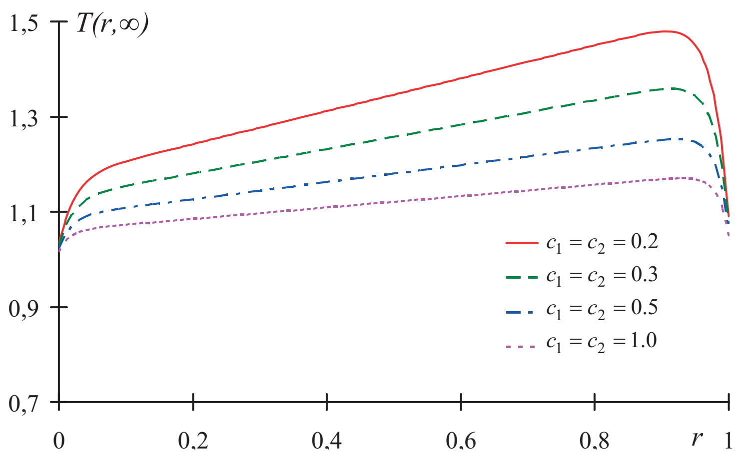

Fig. 16. Steady temperature distributions on the contact surface of the linings.

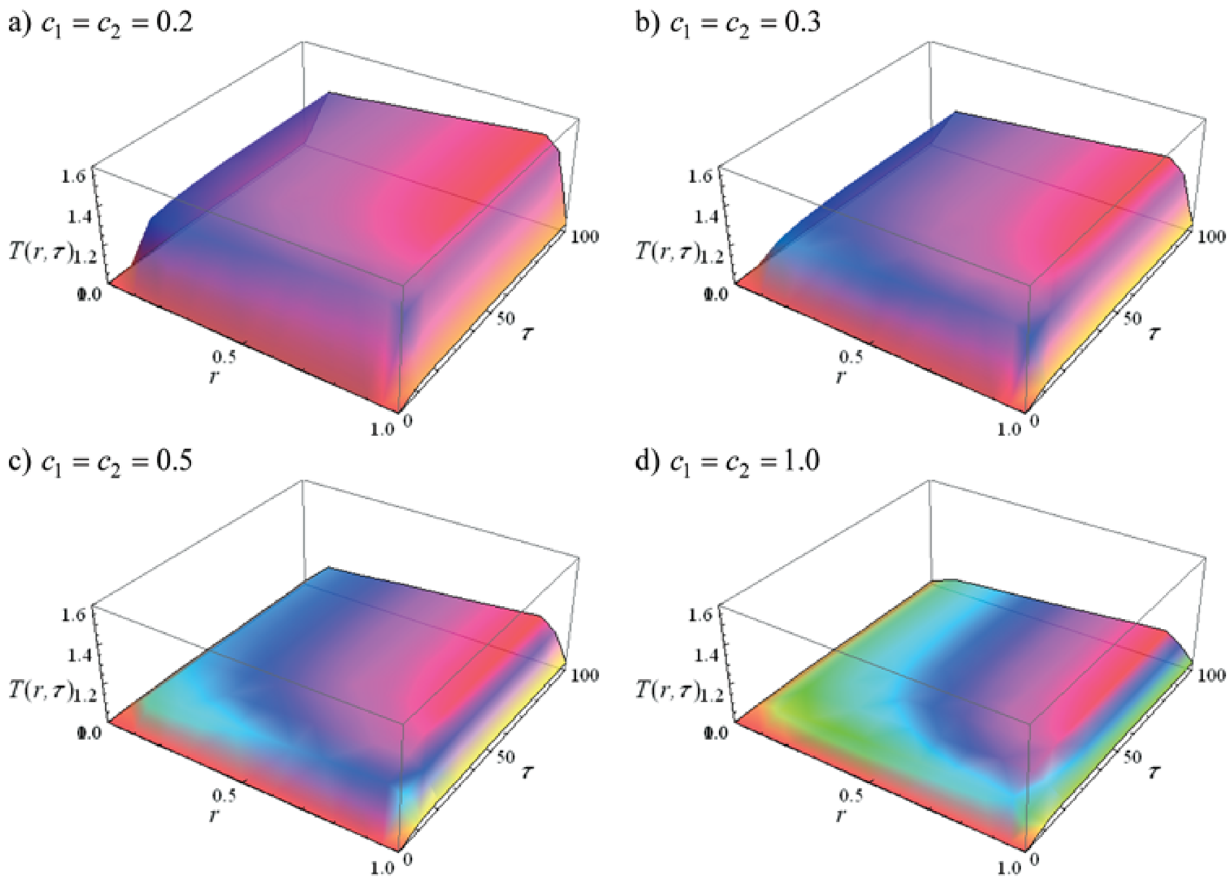

Fig. 17. Time evolutions of the temperature distributions on the surface of the linings.

In this case the temperature distributions achieve a steady state. Inside the contact surface of the linings temperature changes linearly with the radius r, whereas at the boundaries of contact of the linings temperature is significantly lower. For higher values of c_1 and c_2, responsible for heat conductivity of the linings, temperatures in a steady state are lower than for lower values of these parameters.

In Figure 17 the time evolutions of the previously obtained steady temperature distributions are presented.

The numerical analysis of the mathematical model describing thermal phenomena in a friction clutch has been also carried out for the case of the steady pressure distribution on the contact surface of the clutch linings with the form $p(r,\tau) = A\ r$, where $A = const$. Figure 18 presents the steady temperature distributions on the surface of the clutch linings for different values of c_1 and c_2 parameters.

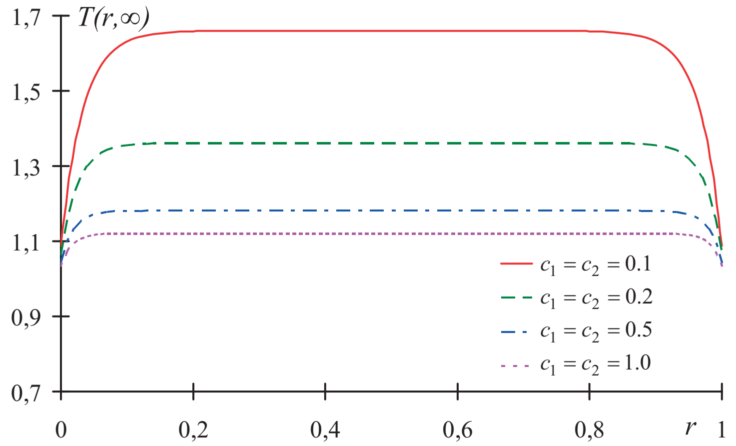

Fig. 18. Steady temperature distributions on the contact surface of the linings.

The temperature distributions achieve the steady state due to establishment of thermal equilibrium between the heat generated in the clutch and the heat transferred to the environment of the linings, i.e. to the clutch discs and to a medium surrounding them (e.g. the air). Inside the contact surface of the linings temperature is constant independently of the radius r, whereas at the boundaries of contact of the linings temperature is significantly lower. This decrease of temperature at the boundaries of contact of the linings results from heat exchange between the linings and the surrounding air. Moreover, for higher values of c_1 and c_2 parameters, characterizing heat conductivities of the linings in the dimensionless form, temperatures in a steady state are lower than for lower values of these parameters. It results from higher heat propagation speed in the linings, which makes the heat energy flow away quicker from the inner part of the contact to its boundaries, where the heat is given up to the environment.

7. Recapitulation and conclusions

The presented work is devoted to investigations of tribological phenomena and processes occurring on the contact surface of linings of a mechanical friction clutch. The results results concern both the processes of wear of the friction linings and the processes of heat generation and propagation in the friction clutch as a result of friction. For modelling and computer simulations of wear processes the general non-linear differential model of wear was used, where speed of wear was non-linear function of contact pressure and rubbing speed. Moreover, the integral model of wear taking into account hereditary and memory processes was also used for modelling and computer simulations of wear processes. Non-uniform distribution of generated heat flux, heat conduction of particular friction materials and heat transfer between the friction linings and their environment were taken into consideration in simulations of thermal phenomena in the clutch. The goal of the experimental investigations carried out, was confirmation of the proposed mathematical models describing tribological processes occurring in a mechanical friction clutch. The processes of wear of the clutch friction lining material have been verified experimentally, although the obtained results were compared to the numerical computations obtained for the linear wear model only. However, it managed to prove that a decrease of the moment of friction force transmitted by the clutch at a constant force pressing the discs, or changes of the contact pressure distribution occurs in accordance with the proposed mathematical model. Also the performed simple experimental qualitative verification of the model describing thermal processes in a clutch indicates relatively good qualitative conformity of the numerical solutions with the results of the experimental investigations. In the result of the conducted numerical analysis for a wider range of parameter changes it was possible to determine the non-uniform contact pressure distributions as well as the wear of particular clutch linings on the surface of contacting friction materials for any instant of time. It allows for a better understanding of wear mechanisms of clutch linings which may be used e.g. for strength analysis of systems of this type. Consideration of the contact pressure distribution changes allows more accurate determination of the moment of friction force transmitted by the clutch. The presented model describing thermal phenomena in a clutch enabled determination of the actual temperature distributions on the contact surface of the clutch friction linings, which was confirmed experimentally by means of the simple experiment. Concurrent modelling of friction phenomena, wear processes and processes of heat generation and propagation in a clutch allows therefore a more accurate determination of the moment of friction force, which makes it possible to more accurately predict dynamics of all power transmission system which includes a clutch considered in the work.

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