

# EXPERIMENTAL DETERMINATION OF SUSPENSION MAGNITUDE-FREQUENCY RESPONSES USING ELECTRO-HYDRAULIC ACTUATORS - TESTING AND DATA PROCESSING METHODS

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## Summary

The paper presents the use of magnitude-frequency properties and practically based experimental methods which have been applied to determine them with an electrohydraulic vibration excitor. The possibility to use these characteristics for linear mathematic models was described along with the manner to estimate dynamic characteristics based on the measurements of excitation and response of a physical object. The construction and control of a electrohydraulic vibration excitor is presented in brief and some basic information about the Remote Parameter Control method is also given. Measurement issues related to obtaining valuable signals have been presented with the focus on fundamental relations between the values of displacements and accelerations in various ranges of frequencies and some advice concerning the preparation of signal of test excitation were formulated. The process of measuring signal processing in order to estimate a power spectral density and then, based on these results, the magnitude characteristics of a suspension is also presented. Differences in results obtained during computer simulation for linear model and for possibilities to use presented method in various areas of vehicle dynamics testing were highlighted.

**Keywords:** vehicle dynamics, suspension dynamics, frequency characteristics, electrohydraulic vibration excitators

## 1. Introduction

Designing vehicle suspensions involves solving numerous problems covering many aspects, concerning:

- dynamic suspension properties related to oscillatory motion, shaping comfort and safety limits of vehicles in dynamic conditions,
- kinematic and elastokinematic characteristics responsible for keeping optimal wheel-road surface contact geometry,

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- mechanical properties of suspension material, i.e. strength and fatigue strength,
- as small suspension package space as possible in order to obtain the largest possible passenger and luggage compartment,
- high suspension reliability and durability,
- low manufacturing costs.

As a result, the design process of a relatively simple mechanism, namely a vehicle suspension, is a multi-faceted process with often conflicting limitations.

This article presents a method of experimental tests determining dynamic suspension characteristics mental tests, which is one of the most important aspects of the suspension design process, but still only one of many, as was stated above.

Car suspension is an oscillatory system, affected by various excitations (Fig.1.):

- the most frequent one in terms of vehicle operation is kinematic excitation  $z_w$  caused by the road surface profile,
- another one is force excitation related to inertial forces acting on the vehicle body.

These excitations result in changes in such variables as absolute bounce displacement of sprung mass  $z_M$  and unsprung mass  $z_m$ , and a relative displacement of both the masses, which describe the suspension deflection (rattle space)  $z_M - z_m$ , also the acceleration of both these masses ( $\ddot{z}_M, \ddot{z}_m$ ), and also tyre-road contact forces  $F_d$ . The relation between road excitation and these variables treated as system output (responses) in a function of excitation frequency are the dynamic characteristics of suspension, also called suspension transmissibility functions [3], frequency response functions or magnitude-frequency characteristics (Bode magnitude plot) [5,8,13,14].

A knowledge of frequency responses is essential to assess suspension functioning in terms of comfort and safety, with technical limits, namely a limited suspension travel space, also being an important consideration. Ride comfort is often assessed by use of sprung mass

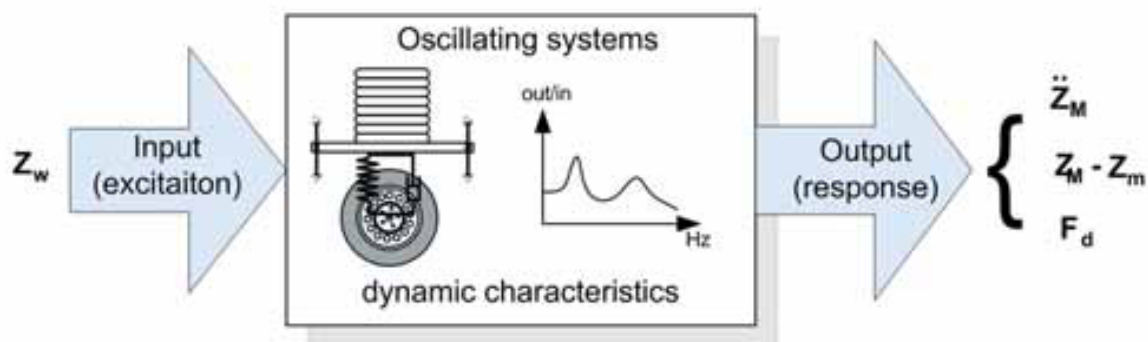


Fig. 1. A quarter car suspension as a dynamic system

acceleration transfer function (acceleration amplification function),  $\bar{Z}_M(\omega)$ , safety – with an analysis of dynamic wheel load amplification function  $F_d(\omega)$ , and by analysis of wheel rattle space amplification function. The latter variable translates into a safety issue when suspension kinematic is being considered, which changes wheel camber and steer angles.

In the case of linear dynamic systems, which enable relatively simple mathematical models to be built amplification functions can be evaluated using fundamental methods of control engineering (exemplary description of using these methods for a linear quarter car model can be found among e.g. in [6,15]).

This is done by determining Fourier transforms of differential motion equations with zero initial conditions and then by determining frequency response functions for selected inputs and outputs. These frequency responses can be expressed in algebraic and exponential form [2]:

$$G(j\omega) = \frac{x(j\omega)}{u(j\omega)} = P(\omega) + jQ(\omega) = A(\omega)e^{j\varphi(\omega)} \quad (1)$$

Gain of frequency response function expressed as:

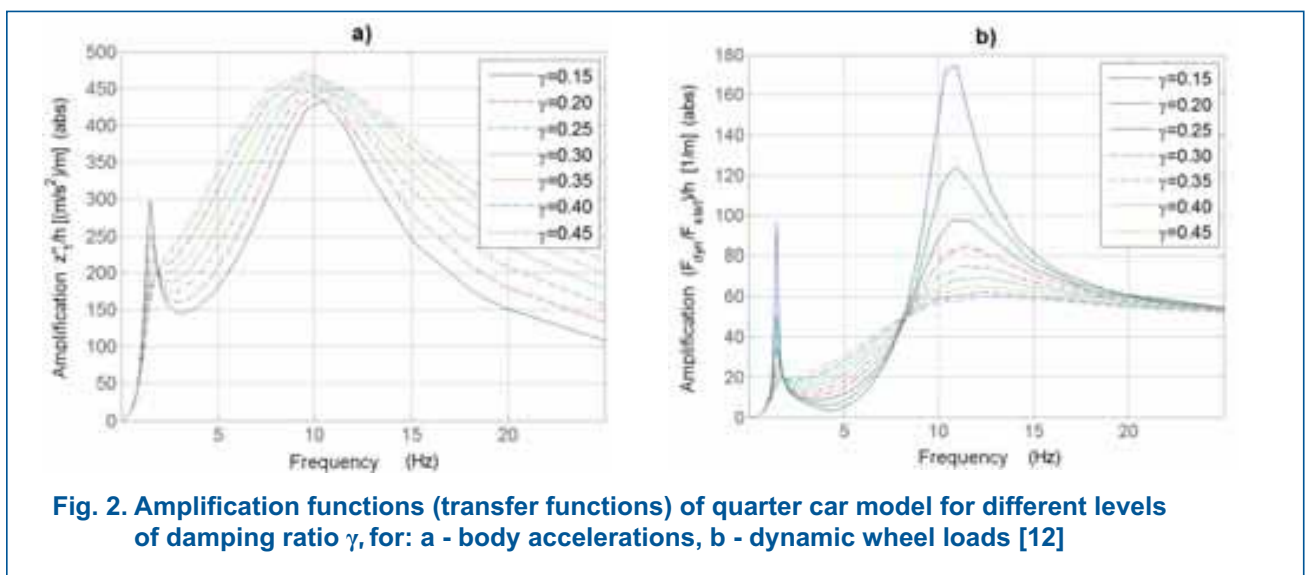
$$A(\omega) = \sqrt{P^2 + Q^2} \Big|_{\omega} = \frac{A_{\text{wyjścia}(\omega)}}{A_{\text{wejścia}(\omega)}} \quad (2)$$

physically means the gain between magnitude of input (excitation) and magnitude of output (response) in form of sinusoidal signal.

Frequency response function argument  $\varphi$  expressed as:

$$\varphi(\omega) = \arctg \frac{Q}{P} \Big|_{\omega} \quad (3)$$

physically means the phase shift of sinusoidal signals – input and output.



With which are commercially available software tools for designing and analysis of control systems and objects, such as Control Toolbox of Matlab computing environment or others, we can relatively simply determine the frequency characteristics for a linear quarter car, as presented in Fig. 2, for various values of damping ratio, defined as:

$$\gamma = \frac{c_1}{2\sqrt{k_1(m_1+m_2)}} \quad (4)$$

where for linear quarter car model  $c_1$  is a shock absorber damping coefficient,  $k_1$  suspension stiffness and  $m_1$  i  $m_2$  are sprung and unsprung masses.

## 2. Evaluation of suspension frequency characteristics based on the analysis of signals from experimental tests

Frequency characteristics of suspension can also be determined experimental method, by analyzing the relations between input signal and output signals measured and acquired during tests on a physical object, namely a quarter car test rig during the tests presented (Fig.6).

The measured signals and construction of the test rig are described in the article below. Here the means of signal analysis will be presented.

During the experiment with a physical object it is possible to measure and acquire input and output signals for the tested object, e.g. vehicle suspension. After preparing signals in the form of time histories, frequency response evaluation can be made using expression (5) [1]:

$$\hat{H}_{xy}(\omega) = \frac{\hat{G}_{xy}(\omega)}{\hat{G}_x(\omega)} \quad (5)$$

where:

$\hat{G}_x(\omega)$  – estimator of power spectral density of input signal,

$\hat{G}_{xy}(\omega)$  – estimator of cross power spectral density of input and output signals.

However, the use of this formula requires knowing the value of power spectrum or power spectral density - PSD (power spectrum value divided by frequency band for which a given PSD value was determined). The two-sided continuous power spectral density can be interpreted as a limit [5]:

$$S_x(\omega) = \lim_{\Delta\omega \rightarrow 0} \frac{\Delta\langle f_n^2(t) \rangle}{\Delta\omega} \quad (6)$$

where:

$\Delta\langle f_n^2(t) \rangle$  – power share of function  $f(t)$  near the frequency  $\omega_n$ ,

$\Delta\omega$  – frequency band for witch power share is determined.

Measurements only help to estimate the power spectrum or power spectrum density, but not to determine them exactly. This is a more complex task, which was determined mainly from a limited set of samples [16].

The estimate of power spectral density can be found with various methods, for example:

- squaring the magnitude of  $f(t)$  – signal Fourier transform and thus obtaining the periodogram [16],
- calculating discrete Fourier transform (DFT) of autocorrelation  $f(t)$  signal function [16].

When calculating the estimates of power spectral density with too few signal samples, unsatisfactory results are obtained which are shown as considerable PSD signal fluctuations. In order to obtain smoothed PSD estimates various method are used, for example calculating and averaging periodograms. One of the most common is the Welch method, which produces in the so-called modified periodogram. It is a modified version of the Bartlett method [16].

Both methods involve dividing the signal (a series of samples) into several periods, for which shorter periodograms are calculated. Next a averaged periodogram an average over all the segments is calculated. The difference between the Bartlett and Welch methods of spectrum estimation is that the first one uses segments which do not overlap, while the other applies overlapping segments and time windows other than rectangular - e.g. the Hanning windows. As a result the Welch method ensures estimation with a smaller variance in comparison with the Bartlett method. This method is programmed among others in a Matlab environment and enables to the power spectral density estimate or cross-spectral density expressed in units of power per radian per sample (rad/Sa) or Hz to be obtained.

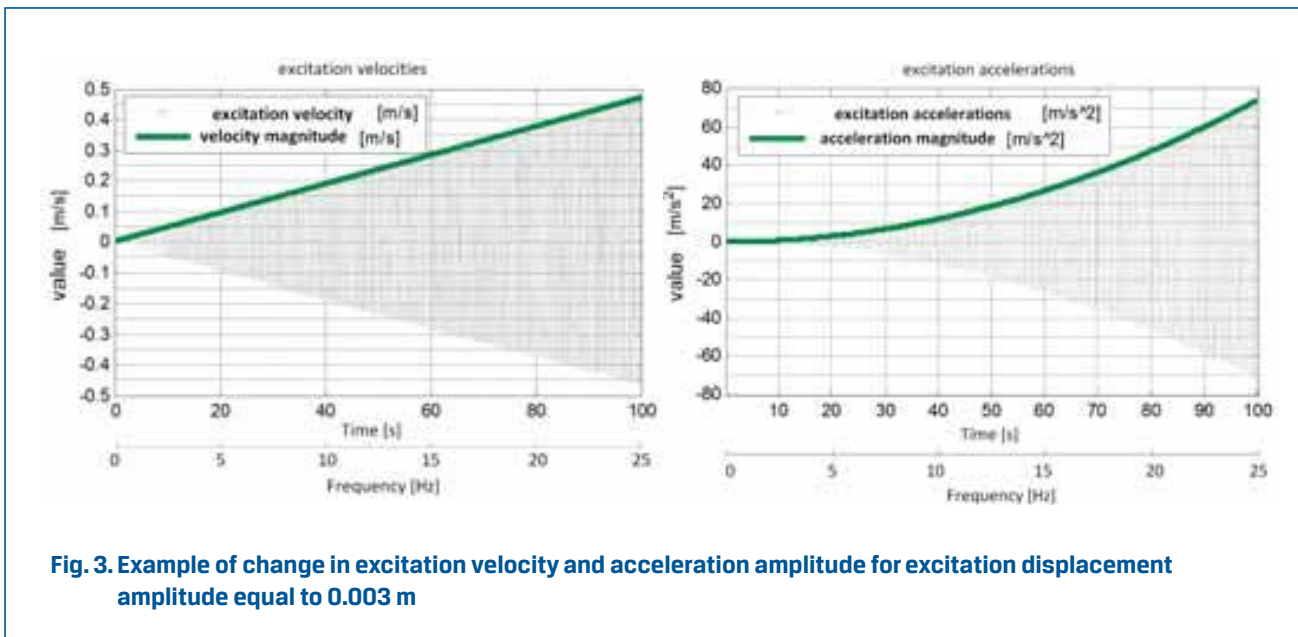
Using calculated estimates of power spectral density we can calculate estimate of the frequency response function or one can use Matlab function for estimating the frequency response estimate using the presented procedure.

### **3. Experimental test for determination of frequency characteristics**

Performing tests aimed at obtaining input and output signals of a physical model of suspension requires methods which enable appropriate excitation signals to be generated and to measure those signals as well as the signals of suspension response.

An important issue in both cases is the quality of generated input signal and the quality of measurements. Both in both case this quality is not perfect due to technical problems and this fact must be taken into consideration when performing the experiment.

One of real problem is noise in the measured signals which stem from nonlinearities of physical suspension and in the quality of measurement system (sensors and system of signal processing and recording).



**Fig. 3. Example of change in excitation velocity and acceleration amplitude for excitation displacement amplitude equal to 0.003 m**

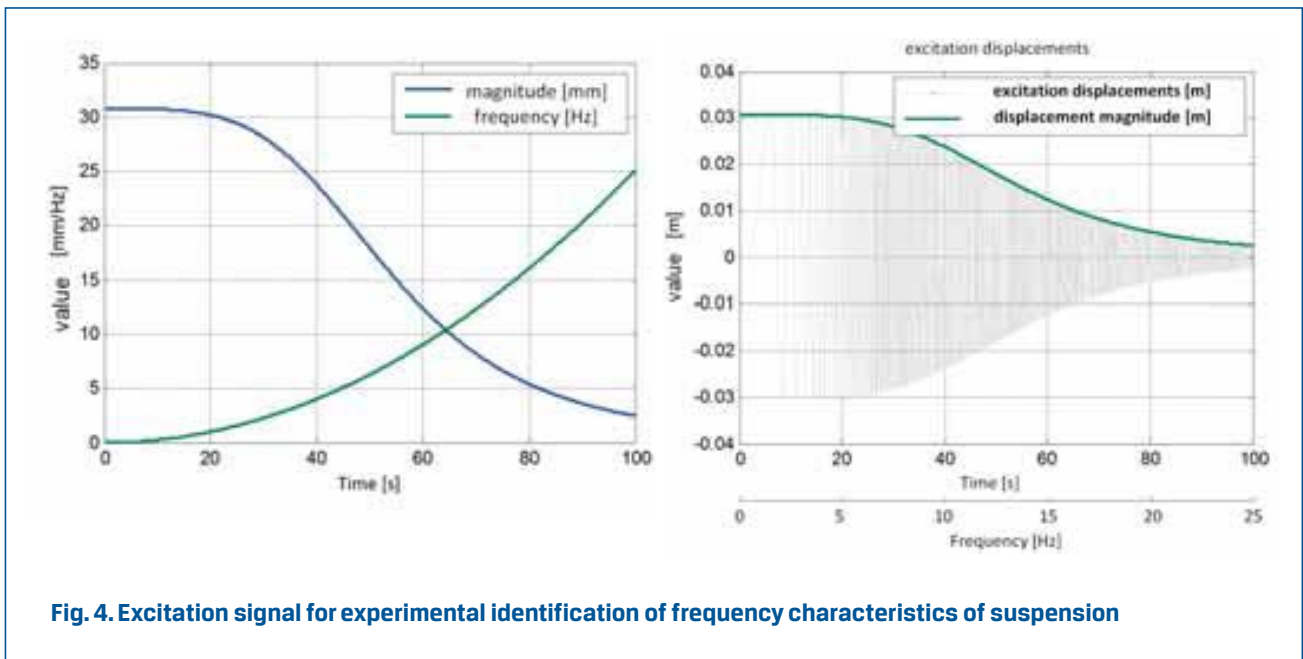
An additional problem is the range of variance of measured variables resulting on the one hand for accelerations from a square relationship between its amplitude and its frequency and of the other hand in displacement from significant filtering of body displacement at higher frequencies. The problem of a considerable variance of measured accelerations is illustrated in Fig. 3., showing how the values of velocity and acceleration of excitation vary with changes in frequency of constant 3 mm magnitude sinusoidal excitation.

Yet another problem is the fact that when using a signal with linearly increasing frequency over time, for lower frequencies we obtain a record of incomplete periods for given frequencies - e.g. at 25 Hz at one second we can record 25 periods but for 0.5 Hz we already need 2 seconds to record one full period of signal.

A wide range of signal fluctuation causes problems with accuracy of the sensors used, resulting from the fact that highly accurate measurement is usually connected with a narrow measurement range and on the other hand a wide measurement range is connected with a significant measurement noise for low values of the measured signal. This applies both to problems with acceleration measurement at low frequencies and to body displacement measurement at high frequencies in the test frequency range. A particular problem is the range of low frequencies where the resonance frequency of the sprung mass lies.

To optimize the excitation signal and account for both these problems a specially developed input signal was generated. This signal takes into account measurement problems and is important for suspension dynamics evaluation frequency range (up to 25 Hz). Changes of the amplitude and frequency of this signal over time and its time history are presented in Fig. 4.

For low frequencies the author decided to perform large displacements as far as was



**Fig. 4. Excitation signal for experimental identification of frequency characteristics of suspension**

technically possible. They were limited, however, by two aspects, i.e. the maximum stroke of excitation actuator piston and a maximum suspension working space, which during resonance was larger than excitation amplitude.

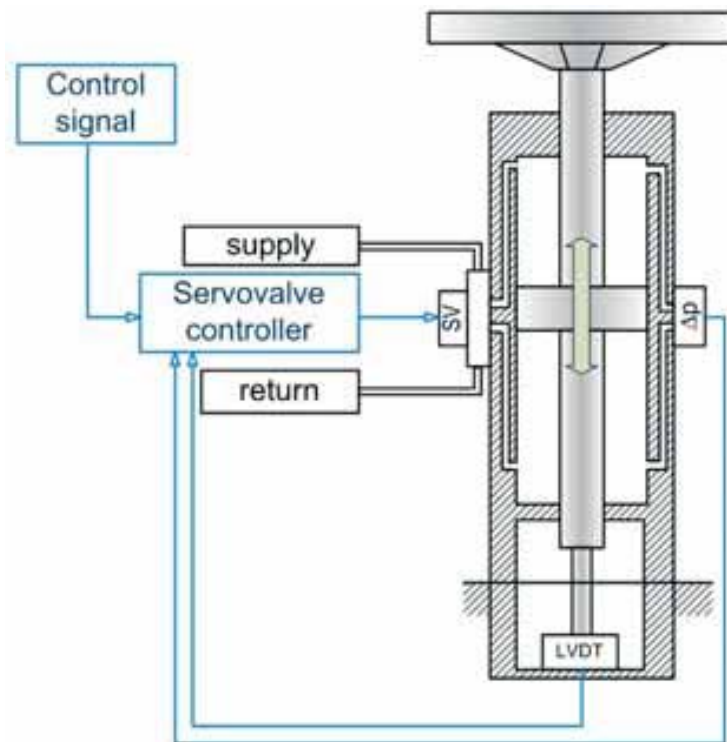
### 3.1. Electro-hydraulic testing system

To conduct experimental tests determining the dynamic characteristics of suspension it is essential to generate kinematic excitation with shapeable amplitude and frequency according to test needs. In the case of excitation with constant amplitude and variable frequency it is possible to obtain such an excitation with test rigs driven by electric motors having crank-shaft mechanism to generate linear excitation. However, if it is also necessary to generate a variable amplitude with a frequency as described above, the most common solution are two side operation hydraulic actuators with electrohydraulic control, also called electrohydraulic vibration excitators [10,11] or servohydraulic shakers. Such hydraulic actuators together with additional elements constitute complete testing systems enabling laboratory simulation tests to be performed of vehicle components or even whole cars. These systems are also called road simulators (tire-coupled or spindle-coupled) or servohydraulic systems or test rigs.

The hydraulic actuator presented in Fig. 5 is controlled with a servovalve controlling a fast flow of oil under the pressure about 20 MPa, into and from the actuator's chambers. The servovalve responds to a control signal sent from the servovalve controller, which generates appropriate control signal using follow-up control with reference to control signal from the upper level closed loop of the control system.

According to test needs a piston rod displacement signal can be used both as a feedback





**Fig. 5. A cutaway view [7] and control scheme of electro-hydraulic shaker [4,9]:**  
 1-piston rod, 2-high pressure inlet, 3-servo-valve case, 4-lower part of piston rod,  
 5-actuator chamber, 6-piston, 7,8-seals

signal or as a control signal - as presented in Fig.5. As a feedback signal a signal acquired from a built-in LVDT transducer can be used or one from an additional force transducer mounted on actuator's piston rod.

Choosing appropriate servo-valve controller's gains is a complicated task, but it is an essential element in ensuring obtaining exact displacements or forces of piston rod.

The difficulty of this task is connected with the choice of controller's gains to overcome the need for short reaction time on the one hand (minimisation of phase lag between control signal and obtained response) and the need for small overshoot to get precise reproduction of the required amplitude and important by the stability of the system work on the other.

Additionally technical limitations of shaker must be considered in control, ones which are associated with maximum stroke, maximum supplying oil pressure and flow.

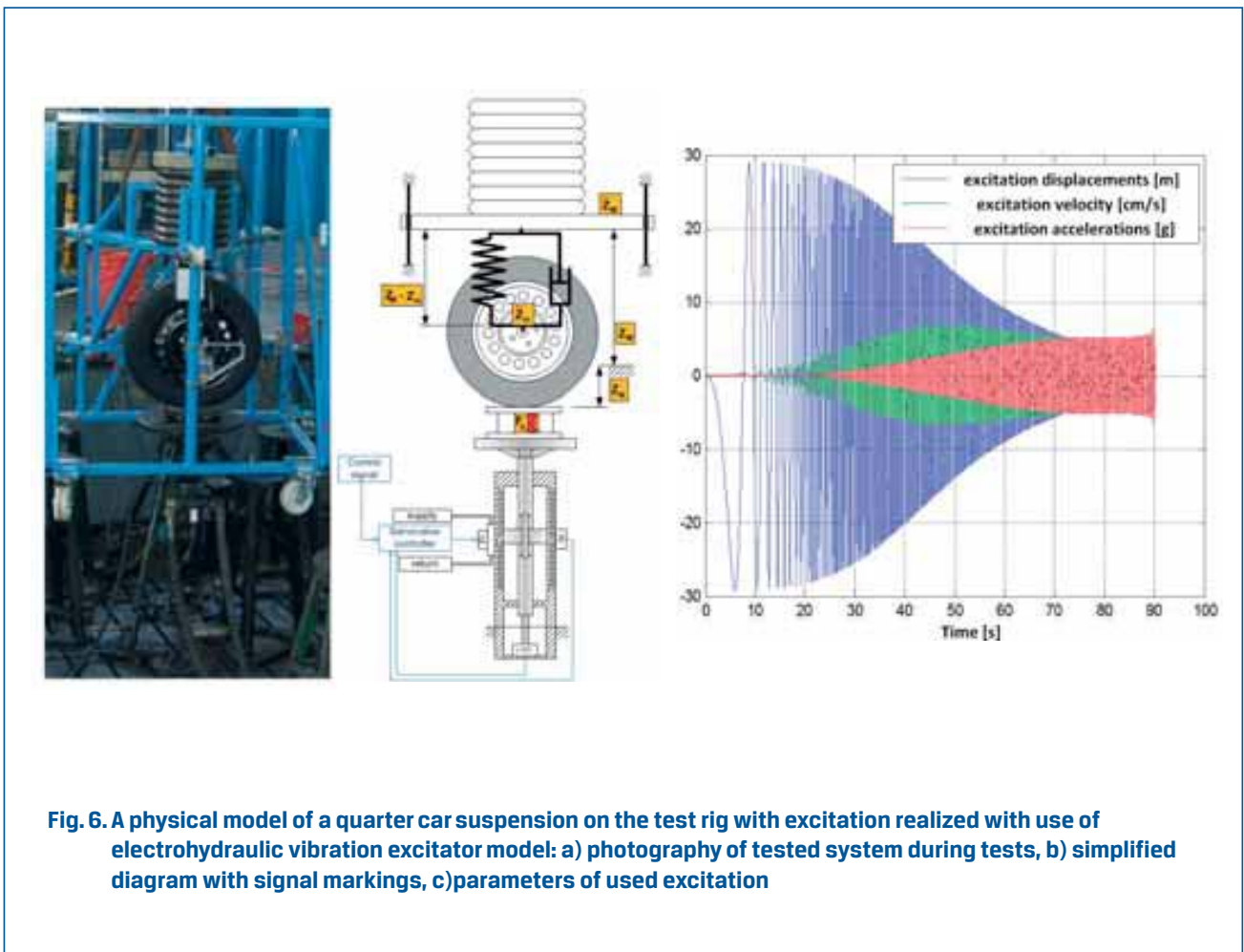
In some cases to improve control quality additional signals are used - for example Fig. 5 presents a diagram of using supplemental signal of differential pressure between actuator chambers, which helps to shorten reaction time of shaker to applied input control signal.



### 3.2. Test rig for determining dynamic characteristics of a quarter car suspension

Using an electrohydraulic vibration excitator as an element generating kinematic excitation it is possible to use it as a source of such excitation - in other words, with some restrictions, as a road simulator. The main restriction concerns the fact that shaker is able to generate only vertical excitation and if it involves a rubber tyre wheel and not directly a wheel or its support, then the differences between the non-rotating and rotating wheel may cause some differences in the behaviour of unsprung and sprung masses despite generating the same kinematic excitations.

The method which is used during tests where vibrations created by kinematic excitations must be the same as in the operation environment of the vehicle is a method of control of kinematic excitation to be obtained in response to a shaker to control signal exactly the same time history of vibration e.g. of body or a wheel as it was during usual operation. This is a simulation method based on reproduced of output signals. This method causes some distortion of the excitation (input signal) in comparison with real excitation but it guarantees the same result of excitation acting on suspension. The complete method used to obtain this goal is called RPC method - Remote Parameter Control by its designers [11].



**Fig. 6. A physical model of a quarter car suspension on the test rig with excitation realized with use of electrohydraulic vibration excitator model: a) photography of tested system during tests, b) simplified diagram with signal markings, c) parameters of used excitation**

This method was used when shaping control signal (Fig.6) which is the working basis for the controller of electrohydraulic vibration excitator's servovalve.

## 4. Data processing – characteristics estimation

During tests on a test rig kinematic excitation is generated according to given time history of control signal generated earlier in RPC process on the base of developed excitation signal.

The measurement of obtained excitation described earlier and presented in Fig. 6 variables (an example of obtained signals in Fig. 7) after appropriate signal processing enable frequency characteristics of a suspension to be produced.

Due to limitations applied at the test rig in the frequency range (which was between 0 and 25 Hz), it was advisable to perform low-pass filtering of recorded signals with a break frequency of 25 Hz.

Signals recorded during tests were processed at the following stages:

- removal of constant component of signal,
- removal of signal trend, if necessary,
- signal low-pass filtering,
- calculation of intermediate variables, based on directly measured signals,
- calculation of the estimate of the frequency response function with a method based on estimates of power spectral density and cross power spectral density,
- draft of the magnitude and the phase of a frequency characteristics.

An example of one of them compared with analytical results for suspension model with identified parameters is presented in Fig. 8.

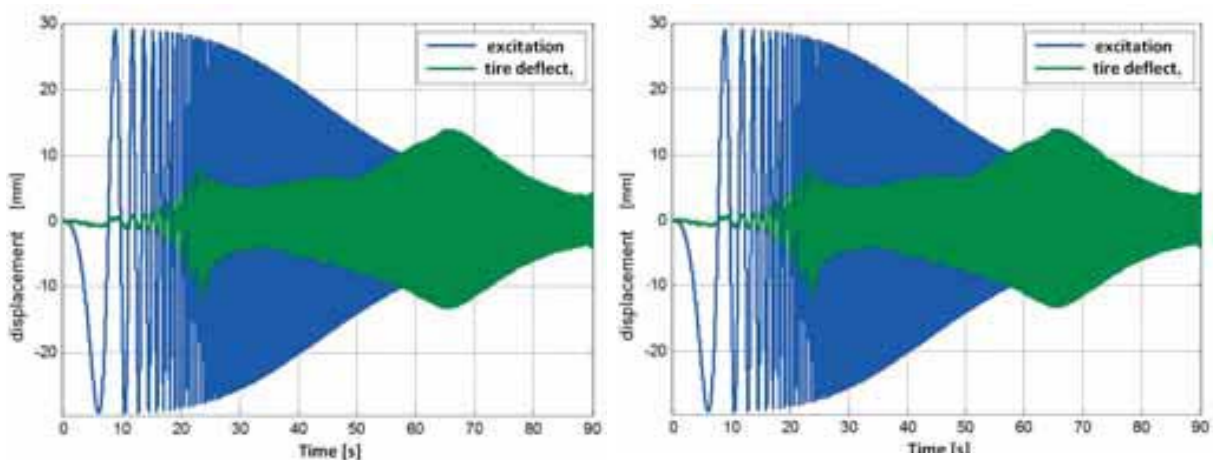
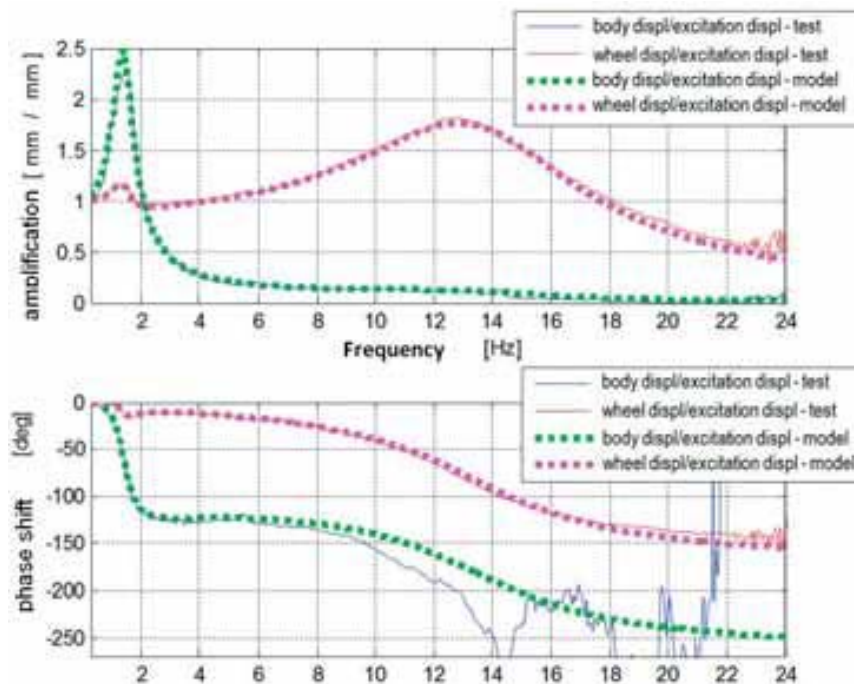


Fig. 7. Examples of direct comparison of measured excitation and response signals in time domain



**Fig. 8. Example of one of frequency characteristics obtained in experimental tests (marked: exp) compared with characteristics determined analytically (marked: model).**

Identification was made for linear model with linear stiffness and damping characteristic and was conducted for the parameter of damping and stiffness; masses were measured at the test rig. These parameters (after rounding) for linear quarter model were  $c_1 = 1900$  Nm/s,  $k_1 = 33$  kN/m,  $k_2 = 290$  kN/m,  $m_1 = 365$  kg and  $m_2 = 41$  kg.

This is the simplest model whose accuracy was discussed among others in paper [7], where the authors highlighted their research results for the frequencies(1-24 Hz) chosen of sinusoidal excitations and based on other works also proved that the linear model gives a reasonable approximation of the unsprung mass amplitude in the tested frequency range and for sprung mass it produces an effect of over predicting amplitude values in a computer simulation in comparison to the measured results. Such an effect was also found when comparison was made in this paper, especially in the resonance range of sprung mass.

As a result of the decreasing value of body displacement with increasing frequency (which is consistent with the purpose and typical characteristics of suspension) and thus increasing relative measurement error both of amplitude values and time lag, the possibility to determine the characteristics of phase lags on the basis of experimental tests is significantly reduced and its shape presents both the characteristics of suspension and sensors with measurement system, which distorts the shape of those characteristics in comparison with characteristics of linear suspension model.

## 5. Summary

The method of experimental determination of suspension frequency characteristics presented in the paper is based on the use of special test rigs and advanced signal processing procedures.

This method, however, conduct experimental tests to be conducted to be conducted the basis for executing two important tasks in model investigations, namely model parameter identification and model verification.

The aspects of experiment realization presented and results analysis shows a number of difficulties in comparison to model investigation, but provide the basis of evaluating dynamics of real object and shows real technical problems, which during model investigation remain unidentifiable. During later practical implementations based on model investigations this fact causes often a lot of implementation failures.

The presented method is also suitable for determining characteristics of nonlinear models or functional models used in simulation methods such as Software In the Loop (SIL), when it is impossible to analytically determine frequency characteristics of investigated model. It should be remembered that in the case of nonlinear models, obtained characteristics may be adequate only for excitations in a range of variability close to that range used during experimental determination of characteristics.

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