THE PROBABILISTIC APPROACH TO THE ANALYSIS OF POWER DISTRIBUTION SYSTEMS

Joanicjusz Nazarko, Member, IEEE
Bialystok Technical University
Poland

Geoffrey Cross
University of Ulster
Northern Ireland, UK

Wojciech Zalewski
Bialystok Technical University
Poland

***Abstract*—Today's electric power distribution systems are complex large-scale systems changing in time and area. Physical phenomena occurring in a distribution system during transmission and distribution of electrical energy are caused by many different processes proceeding in the system and its environment. Quantitative description of the phenomena occurring in the system by analysis of all possible relationships between these phenomena and processes is, in practice, impossible. The principles describing the behavior of random phenomena may be determined by probabilistic and statistical analysis only. The purpose of this paper is to demonstrate the method of probabilistic analysis in radially operated power distribution systems.**

I. INTRODUCTION

Physical phenomena occurring in a power distribution system during distribution and utilization of electrical energy are caused by many different processes proceeding in the system and its environment. Quantitative description of the phenomena occurring in the system by analysis of all possible relationships between these phenomena and processes is, in practice, impossible. The principles describing the behavior of random phenomena may be determined by probabilistic and statistical analysis only.

As is known, power consumption is a time varying process of random nature hence the mathematical model used for system calculations must express quantitatively the probabilistic nature of load variation. In such models the use of random variables is advisable. The parameters of such models are obtained from data acquired by observation of load variation and the application of statistical methods.

The purpose of this paper is to demonstrate the method of probabilistic analysis in radially operated power distribution systems.

Paper SPT PS 24- 06- 0662 accepted for presentation at the IEEE/KTH Stockholm Power Tech Conference, Stockholm, Sweden, June 18-22, 1995

II. BASIC ASSUMPTIONS

The loads at the buses of the power distribution system vary in time as a result of both periodic changes of human activities and the influence of many factors which may be considered to be random phenomena. The theoretical analysis of the process of load generation at the system buses as well as empirical research lead to the conclusion that the density of the joint probability distribution of the active and *reactive load* received at the system bus can be, at a given moment of time, approximated by two-dimensional normal distribution [3]:

$$g(P_i,Q_i)=\frac{1}{2p\sqrt{V[P_i]V[Q_i](1-r^2)}}\exp\left\{-\frac{1}{2\left(1-r^2\right)}\times\right.$$
$$\times\left[\frac{(P_i-E[P_i])^2}{V[P_i]}-2r\frac{(P_i-E[P_i])(Q_i-E[Q_i])}{\sqrt{V[P_i]V[Q_i]}}+\right.$$
$$\left.\left.+\frac{(Q_i-E[Q_i])^2}{V[Q_i]}\right]\right\} \qquad (1)$$

Where:

P_i – active load received at the bus i,
Q_i – reactive load received at the bus i,
$E[P_i]$ – expected value of active load at the bus i,
$E[Q_i]$ – expected value of reactive load at the bus i,
$V[P_i]$ – variance of active load at the bus i,
$V[Q_i]$ – variance of reactive load at the bus i,
r – correlation coefficient between active and reactive load at bus i.

If a joint distribution of two random variables is described by two-dimensional normal distribution, in the general case the probability density of a module of those variables is not submitted to a normal distribution. Nevertheless if one of those variables is much less than the other, the distribution function of a module of those variables can be also approximated by a normal distribution with sufficient precision. The reference [3] indicates that such approximation for a module of complex power is adequate for practical calculations if the quotient of the reactive power to the active power tgφ ≤ 0.54, as is usually fulfilled in power distribution systems.

$$f(S_i)=\frac{1}{\sqrt{2\pi V[S_i]}}\times\exp\left[-\frac{(S_i-E[S_i])^2}{2V[S_i]}\right] \quad (2)$$

Where:

S_i – the module of the complex load received at the bus i,

$E[S_i]$ – the expected value of the module of the complex load at the bus i,

$V[S_i]$ – the variance of the module of the complex load at the bus i.

Taking into consideration the above mentioned assumptions the expected value and the variance of the module of complex power received at system buses can be described by the following equations [1,2,3]:

$$E[S_i]=\sqrt[4]{M^4[S_i]+2\left(A+V[P_i]V[Q_i]-\sigma^2_{P_i,Q_i}\right)} \quad (3)$$

$$V[S_i]=M^2[S_i]+V[P_i]+V[Q_i]-E^2[S_i] \quad (4)$$

where

$$M[S_i]=\sqrt{E^2[P_i]+E^2[Q_i]} \quad (5)$$

and

$$A=E^2[P_i]V[Q_i]+E^2[Q_i]V[P_i]-2E[P_i]E[Q_i]\sigma_{P_i,Q_i} \quad (6)$$

where σ_{P_i,Q_i} is the covariance of active and reactive load received at the bus i.

Load flow computations are a major tool for analysing the performance of power systems. They involve evaluation of active and reactive power flows in the system elements and of bus voltages. For operational engineers power flow is the basis for system calculations and for optimisation of system control. For planing engineers, power flow influences various factors such as transformer and cable ratings, peak load demand periods, and capacitor bank requirements.

The load flow in a radial system is a function of system component connections and loads. Distribution system components are modelled by their equivalent circuits in terms of resistance, reactance, conductance and susceptance. Under balanced conditions a distribution system can be represented by a single phase model. The component interconnections constitute the equivalent circuit of a distribution system. Loads are normally specified by their active and reactive power requirement assuming that they are unaffected by small variations of voltage and frequency during normal steady-state operation. Taking into consideration the above mentioned circumstances the power flow in a radially operated system can be calculated with satisfactory accuracy for practical implementation from the following relationship [4]:

$$\underline{\mathbf{S}}'=\mathbf{D}^T\underline{\mathbf{S}} \quad (7)$$

where:

$\underline{\mathbf{S}}'$ – n×1-dimensional vector of complex power flow in system branches,

$\underline{\mathbf{S}}$ – n×1-dimensional vector of complex power received at system buses,

$\mathbf{D}$ – n×n-dimensional branch-path incidence matrix,

n – the number of system buses (without the root node).

With the assumption of a constant value of the power factor in power distribution systems of medium voltage [4] it is possible in (7) to replace the vector of complex power $\underline{\mathbf{S}}$ received at system buses by a vector of modules of those powers **S**.

$$\mathbf{S}'=\mathbf{D}^T\mathbf{S} \quad (8)$$

In many cases of power distribution system optimisation calculations, active load losses are assumed as a criterion function. The optimisation is to reduce power and energy losses in lines and transformers. An extensive review of optimisation methods in power distribution systems is contained in the reference [4]. Active load losses in a radially operated distribution system can be approximately calculated from the following equation:

$$\Delta P=\frac{1}{U_n^2}\mathbf{S}'^T\mathbf{R}\mathbf{S}' \quad (9)$$

where:

ΔP – total active load losses in the system,

U_n – rated voltage of the power distribution system,

R – n×n-dimensional matrix of system branch resistances.

III. PROBABILISTIC ANALYSIS OF POWER FLOWS AND LOAD LOSSES

It is evident from (8) that power flows in system branches are a linear combination of loads received at the system buses. Taking into consideration results shown in Section II of this paper it is acceptable to assume that power flow in a given branch of the distribution system can be described by normal distribution with the expected value $E[S'_j]$ and the variance $V[S'_j]$. The parameters of the probability distribution of a power flow in branch j are given by the following equations:

$$E[S'_i]=\sum_{j=1}^{n}d_{i,j}E[S_j] \quad (10)$$

$$V\left[S_i^{'}\right]=\sum_{j=1}^{n}d_{i,j}V\left[S_j\right]+\sum_{j=1}^{n}\sum_{k=j+1}^{n}d_{i,j}\sigma_{S_i,S_j} \qquad (11)$$

where:

$E\left[S_i^{'}\right]$ – the expected value of the module of the complex power flowing in the branch i,

$E\left[S_j\right]$ – the expected value of the complex power module received at the bus j,

$V\left[S_i^{'}\right]$ – the variance of the module of the complex power flowing in the branch i,

$V\left[S_j\right]$ – the variance of the complex power module received at the bus j,

σ_{S_i,S_j} – the covariance of modules of complex power received at buses i and j at the same moment of time,

$d_{i,j}$ – the element (i, j) of the matrix **D**.

According to (9) active load losses in the system are a function of the random vector of complex power flow in the system and system branch resistances. For each branch of the system the active load losses can be calculated from the following equation:

$$\Delta P_i=\frac{S_i^{'2}}{U_n^2}R_i \qquad (12)$$

where R_i is the resistance of branch i.

Taking into account (8), (9) and (10) the expected value and variance of active load losses in the given branch i of the system can be stated from the following equations:

$$E\left[\Delta P_i\right]=\frac{R_i}{U_n^2}\left(V\left[S_i^{'}\right]+E^2\left[S_i^{'}\right]\right) \qquad (13)$$

$$V\left[\Delta P_i\right]=\frac{R_i^2}{U_n^4}\left(2V^2\left[S_i^{'}\right]+4V\left[S_i^{'}\right]E^2\left[S_i^{'}\right]\right) \qquad (14)$$

The active power loss in the whole distribution system is the sum of the losses in the particular branches. Hence the expected value and the variance of active load loss in the whole system can be calculated from the following equations:

$$E\left[\Delta P\right]=\sum_{i=1}^{n}E\left[\Delta P_i\right] \qquad (15)$$

$$V\left[\Delta P\right]=\sum_{i=1}^{n}V\left[\Delta P_i\right]+2\sum_{i=1}^{n}\sum_{j=i+1}^{n}\sigma_{\Delta P_i,\Delta P_j} \qquad (16)$$

where $\sigma_{\Delta P_i,\Delta P_j}$ is the covariance of active load losses in branches i and j.

The main difficulty in operational practice is the accurate determination of the loads at receiving buses and the parameters of their probability distribution.

IV. COMPUTATIONAL EXAMPLE

The application of probabilistic approach to the analysis of power distribution systems is demonstrated by solving a radial power flow problem. The load flow study was tested on a simple radial network (Fig. 1). This network includes 17 nodes and 16 branches.

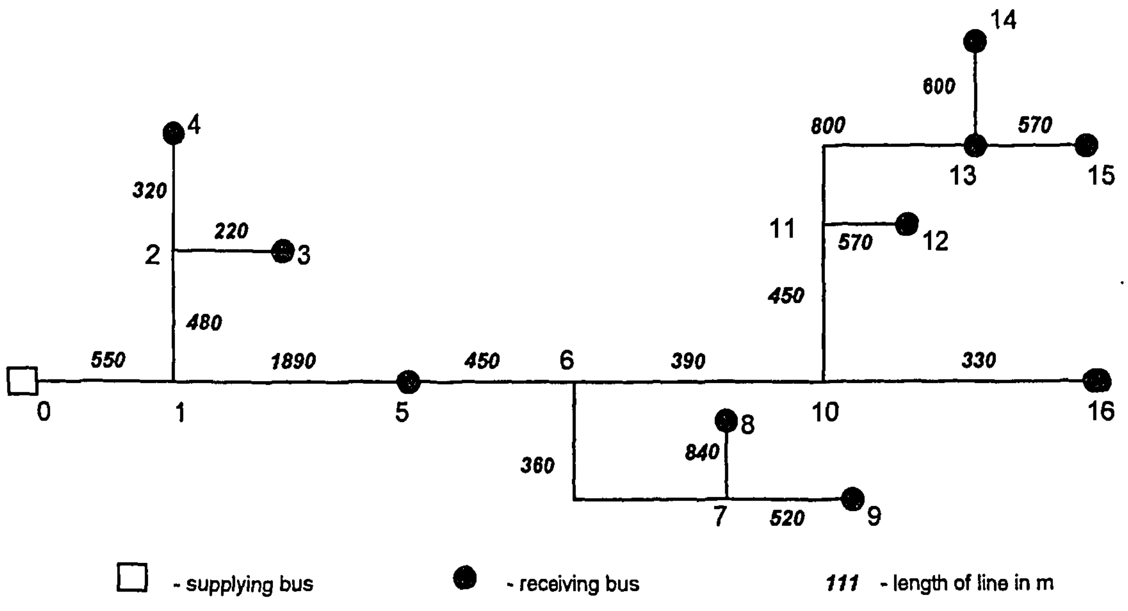

Fig. 1. Diagram of sample radial distribution system of 15 kV

TABLE 1.
ACTIVE AND REACTIVE LOAD RECEIVED AT SYSTEM BUSES

Bus	$E[P_i]$	σ_{Pi} [kW]		$E[Q_i]$	σ_{Qi} [kvar]	
	[kW]	v = 0,1	v = 0,3	[kvar]	v = 0,1	v = 0,3
3	180	18,0	54,0	69	6,9	20,7
4	365	36,5	109,5	146	14,6	43,8
5	94	9,4	28,2	39	3,9	11,7
8	54	5,4	16,2	21	2,1	6,3
9	135	13,5	40,5	54	5,4	16,2
12	300	30,0	90,0	126	12,6	37,8
13	460	46,0	138,0	175	17,5	52,5
14	230	23,0	69,0	92	9,2	27,6
15	390	39,0	117,0	163	16,3	48,9
16	155	15,5	46,5	59	5,9	17,7

TABLE 2.
COMPLEX LOAD RECEIVED AT SYSTEM BUSES

Bus	$E[S_i]$ [kVA]*						σ_{Si} [kVA]					
	v =0,1			v = 0,3			v = 0,1			v = 0,3		
	$r_{Pi,Qi}$			$r_{Pi,Qi}$			$r_{Pi,Qi}$			$r_{Pi,Qi}$		
	0	0,5	1	0	0,5	1	0	0,5	1	0	0,5	1
3	192,99	192,88	192,77	194,77	193,80	192,77	16,98	18,16	19,28	50,71	54,29	57,83
4	393,59	393,35	393,12	397,44	395,34	393,12	34,30	36,88	39,31	102,42	110,24	117,94
5	101,90	101,83	101,77	102,95	102,38	101,77	8,81	9,51	10,18	26,28	28,43	30,53
8	58,01	57,97	57,94	58,55	58,25	57,94	5,09	5,45	5,79	15,19	16,29	17,38
9	145,57	145,49	145,40	147,00	146,22	145,40	12,69	13,64	14,54	37,88	40,77	43,62
12	325,80	325,59	325,39	329,22	327,36	325,39	28,07	30,38	32,54	83,79	90,78	97,62
13	492,71	492,44	492,16	497,20	494,75	492,16	43,42	46,40	49,22	129,69	138,72	147,65
14	248,01	247,87	247,72	250,44	249,12	247,72	21,61	23,24	24,77	64,54	69,47	74,32
15	423,23	422,96	422,69	427,64	425,24	422,69	36,51	39,48	42,27	108,97	117,98	126,81
16	166,03	165,94	165,85	167,55	166,72	165,85	14,63	15,63	16,58	43,70	46,74	49,75

The expected values and standard deviations of active and reactive power demands at each of the receiving nodes are presented in Table 1. It was assumed that the quotient of the reactive power to the active power is closed to 0.4 (tgφ ≈ 0.4). The standard deviations were calculated for two values of variation coefficient (quotient of the standard deviation and the expected value): v = 0,1 and v = 0,3.

The expected values and standard deviations of the modules of the complex power received at the system buses were calculated for three different values of the correlation coefficient of active and reactive load received at the given bus i: $r_{P_i,Q_i} = 0$, $r_{P_i,Q_i} = 0{,}5$ and $r_{P_i,Q_i} = 1$. The results of calculations are presented in Table 2.

On this basis the power flows in each branch of the system were calculated. The standard deviations of the power flow in system branches were calculated for three different values of the correlation coefficient of load at different buses i and j: $r_{S_i,S_j} = 0$, $r_{S_i,S_j} = 0{,}5$ and $r_{S_i,S_j} = 1$. It was assumed that the correlation coefficient of active and reactive load received at the given bus i is equal to 1 ($r_{P_i,Q_i} = 1$). The results of calculations are presented in Table 3.

As a result the power losses in each branch and the total system losses were calculated. To calculate the standard deviations it was assumed that the correlation coefficient of load at different buses equals 1 ($r_{S_i,S_j} = 0$). The results of calculations are presented in Table 4 and 5.

This example is used here only to illustrate the output from calculations where the probabilistic approach is applied. The effect of altered values of variation coefficient and coefficient of correlation between different quantities can be clearly seen in the calculation results.

The probability concept underlines the influence of uncertainty in input data on the outcome of the calculations and indicates the possible effect on the accuracy of the power distribution systems analysis.

TABLE 3.

COMPLEX LOAD FLOW IN SYSTEM BRANCHES

Branch	$E[S_i']$ [kVA]						$\sigma_{S'i}$ [kVA]					
	v = 0,1			v = 0,3			v = 0,1 $r_{Pi,Qi}=1$			v = 0,3 $r_{Pi,Qi}=1$		
	$r_{Pi,Qi}$			$r_{Pi,Qi}$			$r_{Si,Sj}$			$r_{Si,Sj}$		
	0	0,5	1	0	0,5	1	0	0,5	1	0	0,5	1
0 - 1	2547,84	2546,33	2544,81	2572,75	2559,19	2544,81	91,77	148,73	189,27	275,31	446,20	567,80
1 - 2	586,57	586,23	585,89	592,21	589,14	585,89	43,78	47,92	51,72	131,35	143,75	155,16
2 - 3	192,99	192,88	192,77	194,77	193,80	192,77	19,28	19,28	19,28	57,83	57,83	57,83
2 - 4	393,59	393,35	393,12	397,44	395,34	393,12	39,31	39,31	39,31	117,94	117,94	117,94
1 - 5	1961,26	1960,09	1958,92	1980,54	1970,05	1958,92	80,65	120,30	149,80	241,96	360,90	449,39
5 - 6	1859,37	1858,26	1857,15	1877,59	1867,68	1857,15	80,01	115,86	142,99	240,03	347,58	428,97
6 - 7	203,58	203,46	203,34	205,55	204,48	203,34	15,65	16,94	18,14	46,96	50,83	54,43
7 - 8	58,01	57,97	57,94	58,55	58,25	57,94	5,79	5,79	5,79	17,38	17,38	17,38
7 - 9	145,57	145,49	145,40	147,00	146,22	145,40	14,54	14,54	14,54	43,62	43,62	43,62
6 - 10	1655,79	1654,80	1653,81	1672,04	1663,20	1653,81	78,46	107,03	129,44	235,39	321,08	388,31
10 - 11	1489,75	1488,86	1487,96	1504,50	1496,47	1487,96	76,69	99,73	118,37	230,07	299,19	355,10
11 - 12	325,80	325,59	325,39	329,22	327,36	325,39	32,54	32,54	32,54	97,62	97,62	97,62
11 - 13	1163,95	1163,26	1162,57	1175,28	1169,11	1162,57	69,44	83,64	95,76	208,33	268,68	287,27
13 - 14	248,01	247,87	247,72	250,44	249,12	247,72	24,77	24,77	24,77	74,32	74,32	74,32
13 - 15	423,23	422,96	422,69	427,64	425,24	422,69	42,27	42,27	42,27	126,81	126,81	126,81
10 - 16	166,03	165,94	165,85	167,55	166,72	165,85	16,58	16,58	16,58	49,75	49,75	49,75

TABLE 4.

LOSSES OF ACTIVE LOAD IN SYSTEM BRANCHES

Branch	$E[\Delta P_i]$ [kW]						$\sigma_{\Delta Pi}$ [kW]					
	v = 0,1 $r_{Pi,Qi}=1$			v = 0,3 $r_{Pi,Qi}=1$			v = 0,1 $r_{Pi,Qi}=1$			v = 0,3 $r_{Pi,Qi}=1$		
	$r_{Si,Sj}$			$r_{Si,Sj}$			$r_{Si,Sj}$			$r_{Si,Sj}$		
	0	0,5	1	0	0,5	1	0	0,5	1	0	0,5	1
0 - 1	13,50	13,53	13,56	13,65	13,90	14,16	0,973	1,578	2,009	2,927	4,766	6,093
1 - 2	0,63	0,63	0,63	0,66	0,66	0,67	0,093	0,102	0,110	0,283	0,311	0,336
2 - 3	0,03	0,03	0,03	0,03	0,03	0,03	0,006	0,006	0,006	0,019	0,019	0,019
2 - 4	0,19	0,19	0,19	0,20	0,20	0,20	0,038	0,038	0,038	0,115	0,115	0,115
1 - 5	27,51	27,57	27,62	27,88	28,40	28,91	2,262	3,376	4,206	6,810	10,205	12,765
5 - 6	5,89	5,90	5,91	5,98	6,08	6,19	0,507	0,734	0,906	1,525	2,219	2,751
6 - 7	0,06	0,06	0,06	0,06	0,06	0,06	0,009	0,009	0,010	0,026	0,029	0,031
7 - 8	0,01	0,01	0,01	0,01	0,01	0,01	0,002	0,002	0,002	0,007	0,007	0,007
7 - 9	0,04	0,04	0,04	0,05	0,05	0,05	0,008	0,008	0,008	0,026	0,026	0,026
6 -10	4,05	4,06	4,06	4,12	4,19	4,26	0,383	0,523	0,633	1,156	1,583	1,923
10 -11	3,78	3,79	3,80	3,86	3,93	3,99	0,389	0,506	0,601	1,174	1,532	1,826
11 -12	0,23	0,23	0,23	0,25	0,25	0,25	0,046	0,046	0,046	0,140	0,140	0,140
11 -13	4,11	4,12	4,12	4,23	4,31	4,34	0,490	0,590	0,676	1,479	1,918	2,054
13 -14	0,14	0,14	0,14	0,15	0,15	0,15	0,028	0,028	0,028	0,086	0,086	0,086
13 -15	0,39	0,39	0,39	0,42	0,42	0,42	0,077	0,077	0,077	0,237	0,237	0,237
10 -16	0,03	0,03	0,03	0,04	0,04	0,04	0,007	0,007	0,007	0,021	0,021	0,021

TABLE 5.
TOTAL LOSSES OF ACTIVE LOAD IN SYSTEM

	$v = 0,1 \quad r_{Pi,Qi}=1$			$v = 0,3 \quad r_{Pi,Qi}=1$		
	$r_{Si,Sj} = 0$	$r_{Si,Sj} = 0,5$	$r_{Si,Sj} = 1$	$r_{Si,Sj} = 0$	$r_{Si,Sj} = 0,5$	$r_{Si,Sj} = 1$
$E[\Delta P]$ [kW]	60,60	60,72	60,84	61,58	62,69	63,73
	$v = 0,1 \quad r_{Pi,Qi} = 1 \quad r_{Si,Sj} = 0$			$v = 0,3 \quad r_{Pi,Qi} = 1 \quad r_{Si,Sj} = 0$		
	$r_{\Delta Pi,\Delta Pj} = 0$	$r_{\Delta Pi,\Delta Pj} = 0,5$	$r_{\Delta Pi,\Delta Pj} = 1$	$r_{\Delta Pi,\Delta Pj} = 0$	$r_{\Delta Pi,\Delta Pj} = 0,5$	$r_{\Delta Pi,\Delta Pj} = 1$
$\sigma_{\Delta P}$ [kW]	2,62	4,19	5,32	7,90	12,64	16,03

V. CONCLUSIONS

It is seen from the considerations and relationships mentioned above that the analytical determination of values of the basic parameters of probability distributions describing both the power flows and load losses in the system branches and the whole system may be difficult for real power distribution systems. A knowledge is required of many statistical characteristics describing the properties of loads and of elements of the power distribution system and many simplifying assumptions are required on load properties and on forms of probability distribution.

The deficiency of measured data to be used in the load models is often very apparent in distribution systems. In order to express the uncertainty in such situations the use of the statistic compensation of the deficiency of measurement is applied.

It must be noted that although the probability theory provides a good framework for analysis of power distribution systems, the practical application of models presented in this paper needs extensive sets of the statistical data from the past or alternatively some subjective probability distributions for the data concerned must be assigned [5]. In many cases there is no historical or empirical data available. Also the concept of subjective probability may be somewhat difficult for utility engineers to understand. However, the probabilistic approach assists in providing better comprehension of the processes occurring in distribution systems and in improvement of the quality of the input data used in system calculations.

REFERENCES

[1]. Anders G. J.: *Probability Concepts in Electric Power Systems.* John Wiley and Sons, Inc., 1990.

[2]. *Benjamin J. R., Cornell C. A.: Probability, Statistics, and Decision for Civil Engineers.* McGraw-Hill, Inc., 1970.

[3]. Fokin J. A.: *Verojatnostno-statisticeskije metody v rascetach sistem energosnabženija* (in Russian). Moskva, Energoatomizdat, 1985.

[4]. Kujszczyk S.: *Nowoczesne metody obliczeń elektroenergetycznych sieci rozdzielczych* (in Polish). Warsaw, WNT, 1984.

[5]. Nazarko J.: *Modeling of Electrical Power Distribution Systems.* Bialystok, Bialystok Technical University Publisher, 1993.

BIOGRAPHIES

Joanicjusz Nazarko was born in Michalowo, Poland in 1954. He received his Ph.D. and D.Sc. degrees in Electrical Engineering from the Warsaw University of Technology in 1983 and 1992, respectively. He is currently an Associate Professor of Electrical Engineering at the Bialystok Technical University, Poland and he serves as the head of the Division of Informatics, Control and Management in Electrical Power Engineering. His research activity is centered on automation of power distribution systems with emphasis on modelling and analysis of distribution systems in uncertain conditions. Specific research areas have included load estimation, supply restoration, energy loss evaluation and voltage quality. He is a member of IEEE.

Geoffrey Cross received his Ph.D. degree in Electrical Engineering from CNAA, UK in 1980. He is presently a Senior Lecturer at the University of Ulster School of Electrical and Mechanical Engineering, UK. He serves as a Divisional Coordinator for Curriculum Development and Quality Assurance. His research interests are concentrated in the area of software applications to power systems analysis and design and power electronic loads, including harmonic effects and the application of expert systems. He is an external examiner, Ph.D. and M.Sc. in Electrical Power and Power Electronics subjects at a number of UK Universities. He is a member of IEE.

Wojciech Zalewski was born in Bialystok, Poland in 1962. He graduated from the Bialystok Technical University with M.Sc. degree in Electrical Engineering in 1988. He is presently an Assistant at the Faculty of Electrical Engineering at the Bialystok Technical University. He is an currently completing his Ph.D. dissertation in Electrical Engineering at the Warsaw University of Technology, Poland. His research interest areas are the application of expert systems, and probability and fuzzy concepts to power distribution systems modelling and analysis with emphasis on load estimation.