The Fuzzy Approach to Energy Losses Calculations in Low Voltage Distribution Networks

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Abstract: The occurrence of electric energy losses is connected with the energy production, transmission and distribution processes. The proper computing of electric energy losses is one of the most complex problems in power distribution system analysis, demanding consideration of many factors. The accessibility and credibility of data used in calculations is of the greatest importance here. The careful analysis of system losses is desirable in this respect. This article presents the mathematical model of electric energy losses in low voltage networks with application of fuzzy sets theory. Usage a fuzzy approach makes possible to improve loss calculations accuracy. Theoretical statement is illustrated by an example which corresponds to Polish distribution system.

Keywords: Energy losses, Fuzzy sets, Power distribution systems

I. INTRODUCTION

Electric energy losses occurring in electric power networks are essential element of every power utility balance. In Polish conditions they reach a dozen or so per cent, in extreme cases they may gain several dozen per cent the whole energy flowing through all network levels [5, 8]. The biggest participation in arising of electric energy losses belongs to medium and low voltage distribution networks, which determine about 80 per cent all of arising losses [5].

Limiting of energy losses costs occurring in distribution power networks is the potential source of significant savings. The fundamental problem is correct estimation of different kinds of energy losses taking place in distribution networks. The results and the precision of electric energy loss calculation depend mainly on types and the quality of available input data.

Energy losses in municipal distribution power networks are being considered in this paper. In Poland they deliver energy to living, commerce, service and industry consumers situated in the urban areas. They consist of low and medium voltage networks. The low voltage networks are fed from 15/0.4 kV transformers. They are mostly designed as a cable ones. There are low voltage overhead lines in peripheral quarters and detached houses estates.

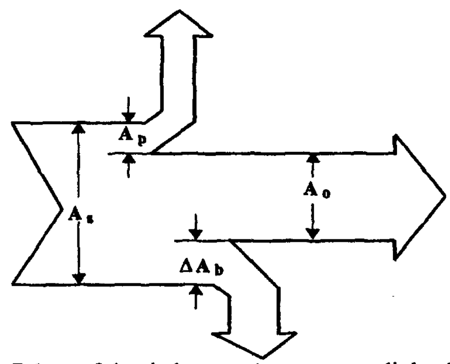

Fig. 1. Balance of electrical energy: A_s - energy supplied to the network, A_p - energy transferred to other distribution areas, A_o – energy sold to consumers, ΔA_b – balance losses of electrical energy

II. CLASSIFICATION OF ENERGY LOSSES

From the point of view of a distribution utility it is useful to introduce a division of electric energy losses into: balance, technical, and trade losses.

Balance losses (ΔA_b) are calculated as a difference between energy supplied to the network and energy recorded as sold to consumers or transferred into other distribution areas (Fig.1).

$$\Delta A_b = A_s + (A_o - A_p) \qquad (1)$$

where:

ΔA_b – balance losses of electrical energy,
A_s – electric energy supplied to the network,
A_o – electric energy sold to consumers,
A_p – electric energy transferred to other distribution areas.

Technical losses (ΔA_t) are physical electric energy losses arising in electric power network elements and connected with electric energy transmission and distribution processes. Trade losses (ΔA_h) are difference between balance losses and technical losses calculated for the network.

Therefore, following equation can be written:

$$\Delta A_b = \Delta A_t + \Delta A_h \qquad (2)$$

where:

ΔA_b – technical electric energy losses,
ΔA_t – technical electric energy losses,
ΔA_h – trade electric energy losses.

III. TECHNICAL ENERGY LOSSES IN LOW VOLTAGE DISTRIBUTION NETWORKS

Network losses can be separated into so-called fixed losses and variable losses. The fixed losses are those occurring due to the magnetisation currents of such items as transformers and reactors. The variable losses are those caused by the flow of current trough the different elements of equipment in the network, and are also termed copper losses. The energy from electrical power loss is converted to heat that tends to increase the temperature of associated electrical component [3].

Technical losses in medium and low voltage distribution networks consist of: load losses in overhead and cable medium voltage lines, distribution transformer windings, overhead and cable low voltage lines, low voltage services, internal feeder lines and idling losses in voltage coils of the meters, medium voltage lines and distribution transformer cores.

Low voltage distribution networks are operated radially. The elementary segment of the network considered in this paper is the fragment consisting of one 15/0.4 kV transformer substation, overhead or cable feeder lines coming out from it, internal feeder lines, electric energy meters and energy capacitors installed by the side of low voltage (Fig. 2).

The total technical losses of electric energy, for one distribution network section including one 15/0.4 kV distribution transformer with low voltage lines and consumers, consist of [5, 8]:

$$\Delta A_t = \Delta A_{Fe} + \Delta A_{Cu} + \Delta A_u + \Delta A_L + \Delta A_{wlz} + \Delta A_{Lc} + \Delta A_k \quad (3)$$

where:

ΔA_t – technical electric energy losses in low voltage distribution network,
ΔA_{Fe} – electric energy losses in transformer iron core,
ΔA_{Cu} – load energy losses in transformer,
ΔA_u – electric energy no-load losses in low voltage lines,
ΔA_L – electric energy load losses in low voltage lines,
ΔA_{wlz} – electric energy losses in internal feeder lines,
ΔA_{Lc} – electric energy losses in meters,
ΔA_k – dielectric losses of energy in capacitors of the low voltage network.

All of the components of technical losses, for considered system, can be calculated as shown by equations below:

$$\Delta A_{Fe} = \frac{\Delta P_{FeN} T_m U_s^2}{U_N^2} \quad (4)$$

where:

ΔP_{FeN} – nominal no-load losses in the transformer core,
T_m – time of the transformer load duration,
U_s – average voltage on the transformer buses,
U_N – nominal network voltage.

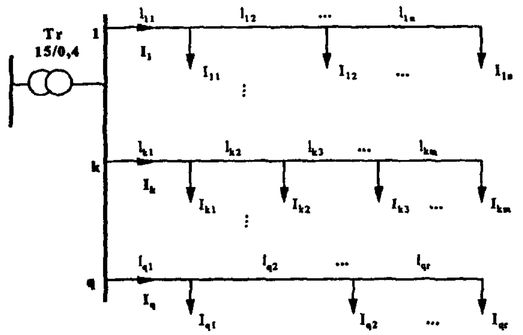

Fig. 2. The exemplary segment of the electric energy distribution low voltage network fed by one transformer substation: $l_{11}, \ldots, l_{qr}$ – lengths of individual feeder lines sections; $I_{11}, \ldots, I_{kr}$ – load currents in individual receiving buses; $I_1, \ldots, I_q$ – load currents coming into individual feeder lines; k – the number of feeder lines coming out of the transformer substation; n, m, r – the number of receiving buses in individual feeder lines

$$\Delta A_{Cu} = \Delta P_{CuN} B^2 \tau_m k_T \quad (5)$$

where:

ΔP_{CuN} – nominal load losses of transformer,
B – transformer load rate,
τ_m – time of maximal losses duration,
k_T – temperature coefficient.

$$\Delta A_u = c_u L T_m \quad (6)$$

where:

L – the total length of low voltage lines fed by considered transformer substation,
c_u – coefficient of unit line losses.

$$\Delta A_L = 3\tau_m \sum_{i=1}^{n} I_{maxi}^2 R_i \quad (7)$$

where:

I_{maxi} – maximal current flowing in *i–th* line,
R_i – resistance of *i–th* feeder line,
n – number of feeder lines,
τ_m – time of maximal load losses duration.

$$\Delta A_{wlz} = c_2 A_k \quad (8)$$

where:

A_k – electric energy flowing through considered termination assembly,
c_2 – coefficient of the relation between losses in services and A_k energy.

$$\Delta A_{lc} = (c_1 n_1 + c_3 n_3) T_m \quad (9)$$

where:

c_1 – unit electric energy losses in 1 – phase meters,
n_1 – the number of 1 – phase meters installed,
c_3 – unit electric energy losses in 3 – phase meters,
n_3 – the number of 3 – phase meters installed.

IV. THE SIMPLIFIED NETWORK MODEL FOR ELECTRIC ENERGY LOSSES CALCULATIONS

Load-flow studies in radially operated distribution networks are relatively simple. The main difficulties result from the high number of load points and very limited information on individual points. Usually, the periodic energy consumption at low voltage is only known.

The exemplary segment of an urban distribution low voltage network is being considered (Fig. 2). The system is fed by one transformer substation working with one 15/0,4 kV transformer. Low voltage feeder lines deliver electric energy to individual receiving buses. In case of buildings meant for many families cable joints are used. If we deal with detached houses, individual services or cable joints of separated consumers groups should be applied. Each reception centre is characterised by load current and feeder line's resistance between the centre and the transformer substation.

The simplification of the model consists in transforming the circuit from Fig.2 into the equivalent circuit shown in Fig. 3. The transformation condition is the equity of total power losses in the real feed system and the equivalent one [1]. Apart from that the equivalent load current I_{eq} is being considered to be equal transformer load current I_{Tr}.

Therefore, following equations can be written:

$$I_{eq} = I_1 + I_2 + ... + I_q = \sum_{i=1}^{q} I_i \tag{10}$$

$$R_z = \frac{\sum_{i=1}^{k} I_i^2 R_{iz}}{I_{eq}^2} \tag{11}$$

where:

R_z – equivalent resistance of all feeder lines, by which power losses in equivalent circuits are equal,

I_{eq} – equivalent load current.

Having the information about the value of the electric energy consumed by individual consumers connected to receiving buses (par example on the basis of meters registration) and having the knowledge about the amount of electric energy flowing through the distribution transformer, it is possible to calculate load currents in individual feeder lines, depending on transformer load current, as follows:

$$I_{eqkmax} = \alpha_k I_{Tr} \tag{12}$$

where:

I_{eqkmax} – the equivalent load current flowing through *k–th* feeder line,

I_{Tr} – transformer load current,

α_k – the ratio of *k–th* maximal equivalent current to transformer load current, while:

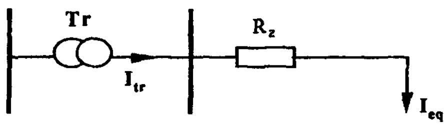

Fig. 3. The equivalent chart of the circuit shown in Fig. 2: R_Z – the equivalent resistance of all feeder lines coming out of the transformer substation; I_{eq} – the equivalent load current; I_{Tr} – the transformer load current.

$$\alpha_k = \frac{I_{eqkmax}}{I_{Tr}} = \frac{a_k \overline{A}}{A_0} = \frac{a_k}{a} \tag{13}$$

where:

a_k – number of consumers fed by *k–th* feeder line,

a – number of all consumers fed by given distribution substation,

$\overline{A}$ – average consumption of electric energy for consumers fed by given transformer substation,

A_o – electric energy flowing through the distribution substation.

V. FUZZY MATHEMATICAL MODEL

Because of significant shortage of data and measurements in municipal distribution electric energy low voltage networks the electric energy losses estimation process in those networks is led in conditions of limited access to information. The information limit concerns individual feeder lines and receiving buses loads and the value of electric energy consumed by specified consumers groups, as well as the technical data of distribution low voltage network elements [5].

The considerations above lead to the conclusion that all the quantities appearing in (3) ÷ (13) are uncertain numbers, being characterised by fuzziness. On the basis of owned information about network parameters, data about consumers and the value of electric energy consumed by individual consumers groups the numbers fuzziness ranges can be estimated [5, 6].

The theory which enables efficient description of unreliable and inaccurate data, and relationship between them, is fuzzy set theory [4].

One of the ways of fuzzy quantity descriptions is to show them in the form of triangle type numbers. In this case to every quantity, one real value may be assigned. As a rule, the assigned value is not known or is known approximately. However, it is possible to specify the number range, in which the number may be situated with enough certainty for practical aims. During the ranges limits estimation it is possible to use both available information about average values of quantities considered, and producers catalogue data. The important source of the information may be also the experience and the practice of the technical personnel employed in district dispatch office of an individual distribution supply company [5, 7, 8].

With such assumption all quantities from equations (3 ÷ 13) may be written in the form as follows:

$$W = (w_1, w_2, w_3) \tag{14}$$

where:

w_1, w_2, w_3 – parameters of fuzzy number W.

For this reason individual components of the mathematical model of the electric energy losses in municipal power distribution low voltage networks may be shown in the form of triangle type fuzzy numbers with parameters put together in Table 1.

Total technical electric energy losses in fuzzy form are represented as a sum of individual components of those losses shown as fuzzy ones. Hence the equation to represent the fuzzy model of electrical energy losses in low voltage municipal power distribution networks is:

$$\Delta\tilde{A}_t = \Delta\tilde{A}_{Fe} + \Delta\tilde{A}_{Cu} + \Delta\tilde{A}_u + \Delta\tilde{A}_L + \Delta\tilde{A}_{wlz} + \Delta\tilde{A}_{Lc} \tag{15}$$

In the equation (15) individual symbols signify the same quantities as in formula (3), but shown as fuzzy ones.

TABLE 1

Parameters of electric energy losses in municipal distribution low voltage networks shown in the form of triangle type fuzzy numbers

Quantity	Symbol	The triangle form of fuzzy number
Nominal no-load losses in the transformer iron core	ΔP_{FeN}	$(\Delta P_{FeN} - \delta\Delta P_{FeN}, \Delta P_{FeN}, \Delta P_{FeN} + \delta\Delta P_{FeN})$
Nominal load transformer losses	ΔP_{CuN}	$(\Delta P_{CuN} - \delta\Delta P_{CuN}, \Delta P_{CuN}, \Delta P_{CuN} + \delta\Delta P_{cuN})$
Time of load duration	T_m	$(T_m - \Delta T_m, T_m, T_m + \Delta T_m)$
Relative time of maximal load duration	t_{sm}	$(t_{sm} - \Delta t_{sm}, t_{sm}, t_{sm} + \Delta t_{sm})$
Voltage during maximal load	U_s	$(U_s - \Delta U_s, U_s, U_s + \Delta U_s)$
Average voltage in the substation	U_{sm}	$(U_{sm} - \Delta U_{sm}, U_{sm}, U_{sm} + \Delta U_{sm})$
Power coefficient during maximal load	$\cos\varphi_s$	$(\cos\varphi_{smin}, \cos\varphi_s, \cos\varphi_{smax})$
Average coefficient of power	$\cos\varphi_{sm}$	$(\cos\varphi_{sm,min}, \cos\varphi_{sm}, \cos\varphi_{sm,max})$
Coefficient of transformer windings resistance conversion because of temperature	k_T	$(k_{Tmin}, k_T, k_{Tmax})$
Transformer load rate	B	$(B - \Delta B, B, B + \Delta B)$
Time of maximal losses duration	τ_m	$(\tau_m - \Delta\tau_m, \tau_m, \tau_m + \Delta\tau_m)$
K – th feeder line load current	I_{eqk}	$(I_{eqkmin}, I_{eqk}, I_{eqkmax})$
Length of i – th section of k – th feeder line	l_{ik}	$(l_{ik} - \Delta l, l_{ik}, l_{ik} + \Delta l)$
Energy consumed by k – th line consumers	A_k	$(A_{kmin}, A_k, A_{kmax})$
Coefficient of unit line losses	c_u	$(c_{umin}, c_u, c_{umax})$
Coefficient describing energy losses portion in services	c_2	$(c_{2min}, c_2, c_{2max})$
Energy unit losses in 1 – phase meters	c_1	$(c_{1min}, c_1, c_{1max})$
Energy unit losses in 3 – phase meters	c_3	$(c_{3min}, c_3, c_{3max})$

After summation of all equation (13) components one receives the following fuzzy form of mathematical model of technical electric energy losses in low voltage municipal distribution networks (14):

$$\begin{aligned}\Delta\tilde{A}_t = \{\{\{[&\frac{1}{U_N^2}(\Delta P_{FeN} - \delta\Delta P_{FeN})(T_m - \Delta T_m)(U_{sm} - \Delta U_{sm})^2] + [(\Delta P_{CuN} - \delta\Delta P_{CuN})(\tau_m - \Delta\tau_m)k_{Tmin}(B - \Delta B)^2] + \\ &+ [(T_m - \Delta T_m)c_{umin}(L - \Delta L)] + 3\sum_{i=1}^{n}[(I_{imin})^2(R_i - \Delta R)(\tau_m - \Delta\tau_m)] + [c_{2min}A_{kmin}] + [(n_1 c_{1min} + n_3 c_{3min})(T_m - \Delta T_m)]\}, \\ &\{[\Delta P_{FeN} T_m U_{sm}^2] + [\Delta P_{CuN}\tau_m k_T B^2] + [c_u L T_m] + [(I_i)^2 R_i \tau_m] + [c_2 A_k] + [(n_1 c_1 + n_3 c_3)T_m]\}, \\ &\{[\frac{1}{U_N^2}(\Delta P_{FeN} + \delta\Delta P_{FeN})(T_m + \Delta T_m)(U_{sm} + \Delta U_{sm})^2] + [(\Delta P_{CuN} + \delta\Delta P_{CuN})(\tau_m + \Delta\tau_m)k_{Tmax}(B + \Delta B)^2] + \\ &+ [(T_m + \Delta T_m)c_{umax}(L + \Delta L)] + 3\sum_{i=1}^{n}[(I_{imax})^2(R_i + \Delta R)(\tau_m + \Delta\tau_m)] + [c_{2max}A_{kmax}] + [(n_1 c_{1max} + n_3 c_{3max})(T_m + \Delta T_m)]\}\}\}\end{aligned} \tag{16}$$

VI. NUMERICAL EXAMPLE

In order to test the proposed method for energy losses calculations an example based on a real municipal distribution low voltage network was performed.

The exemplary fragment fed by one distribution transformer of 15/0.4 kV and 400 kVA rating is considered (Fig. 4). Consumers (blocks of flats) by the low voltage side are fed by 4 cable lines YAKY 4x120 operating in a radial system of 605 m total length [8].

A range of fuzziness for each investigated quantity was estimated on the basis of measurements and simulation studies. Triangle type numbers were employed in calculations. Parameters of fuzzy numbers used in calculations are put together in Table 2.

TABLE 2

Assumed parameter values of the fuzzy model of energy losses

Quantity	Range of fuzziness	Quantity	Range of fuzziness
ΔP_{FeN}	[0.986 ; 1.038 ; 1.090]	I_{Tr}	[0.772 ; 0.902 ; 1.033]
	[- 5 % ; + 5 %]		[- 14 % ; + 14 %]
ΔP_{CuN}	[5.108 ; 5.377 ; 5.646]	A_k	[13728 ; 14451 ; 15174]
	[- 5 % ; + 5 %]		[- 5 % ; + 5 %]
T_m	[675.1 ; 696.0 ; 716.9]	C_u	[1.4 ; 1.7 ; 2.2]
	[- 3 % ; + 3 %]		[- 18 % ; + 29 %]
U_{sm}	[0.352 ; 0.371; 0.389]	c_2	[0.0027; 0.003 ; 0.0032]
	[- 5 % ; + 5 %]		[- 10 % ; + 7 %]
$\cos\varphi_s$	[0.895 ; 0.942 ; 0.989]	c_1	[1.4 ; 1.5 ; 1.7]
	[- 5 % ; + 5 %]		[- 7 % ; + 13 %]
k_T	[1.045 ; 1.100 ; 1.155]	c_3	[2.5 ; 3.4 ; 4.1]
	[- 5 % ; + 5 %]		[- 27 % ; + 21 %]

Energy losses calculations were made for two cases according to available information on the peak load of the distribution transformer:

A) peak load of distribution transformer was exactly known (without fuzziness);

B) peak load of distribution transformer was estimated using fuzzy regression model on the basis of kWh consumption information [6].

Fuzzy triangle type numbers represent electric energy losses computed on the basis of presented fuzzy mathematical model (16). Results of the estimation of technical losses of electric energy for considered exemplary segment of urban distribution low voltage network are summarized in Table 3. The final result of the calculations estimates the range, in which the value of electric energy losses is supposed to be situated. The width of the range depends on the fuzziness extent of individual parameters forming energy losses mathematical model. It is clearly seen from obtained results that additional information (e.g. knowledge of the peak load of the distribution transformer) can significantly decrease the range of uncertainty of output results.

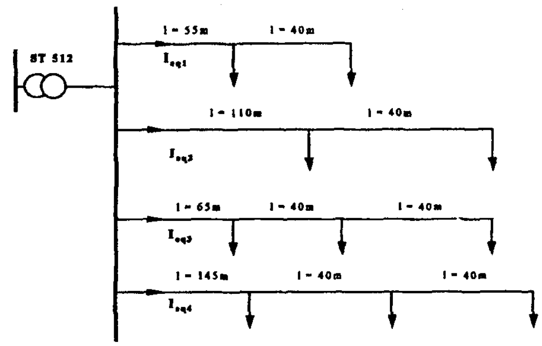

Fig. 4. The exemplary segment of urban distribution low voltage network

TABLE 3

Results of electric energy losses calculations

Study case	Range of fuzziness
Case A	[1039.6; 1.089.1; 1142,5]
	[- 4.5% ; + 4.9 %]
Case B	[1043.3; 1136.8; 1247.2]
	[- 8.2% ; + 9.7%]

To visually display the results of the study the plots of membership function for estimated energy losses were drawn. Fig. 5 presents the graphic image of the fuzziness of the electric energy losses in the considered fragment of distribution network.

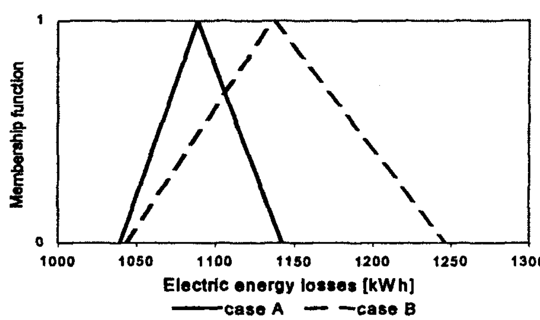

Fig. 5. The graphic image of electric energy losses fuzziness in the considered system

The range of fuzziness of obtained results of energy losses calculations is about twice wider for the case B (when all data is fuzzy) comparing to case A (when peak load of distribution transformer is exactly known).

VII. CONCLUSIONS

In this paper we proposed the new approach to electric energy losses calculations in low voltage power distribution networks. In such systems energy losses estimation is quite complicated because limited information on loads and customers. In order to model system uncertainty, inexactness, and random nature of customers' demand, a fuzzy approach is proposed. The application of fuzzy analysis rules made possible to receive satisfactory estimating results, in spite of the significant measurements shortage occurring in distribution systems. Unreliable and inaccurate input data can be modelled by means of fuzzy numbers.

Considerations presented in the paper confirm the possibility of developing the mathematical model based on the fuzzy sets approach for energy losses estimation in low voltage distribution networks.

The basic advantage of the proposed way of estimation of technical electric energy losses in distribution low voltage networks is an effective use of different kind of available information concerning loads, technical parameters of individual feeder lines and receiving buses, and the energy consumption by customers.

Simulation studies have been performed to demonstrate the efficiency of the proposed scheme on the basis of actual data obtained at real distribution system.

VIII. ACKNOWLEDGMENTS

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