Local entropy patterns in continuous cellular automaton models

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Abstract

The article presents the results of the simulation of a cellular automaton model whose cells assume continuous values. The described experiment examined the behaviour of local entropy and showed the occurrence of its three qualitatively different images with their character resembling Turing patterns. The hypothesis, according to which the behaviour of local entropy might be identified with an image created in a reaction-diffusion process, was presented.

Problem

The research study concerning the mathematical foundations of creating cultures (Zgrzebnicki, in press-a) considers a continuous cellular automaton model as a hypothetical framework. Although there have been many studies on the properties of discrete cellular automata published so far, the models whose cells assume any values limited only by floating point precision have been much less discussed in scientific works. The research presented herein assumed that the discussed automaton may change values of its cells according to any function transforming the values of neighbours in the neighbourhood of a certain radius, and that this radius and weight of the neighbours' contribution to a cell's change may also be arbitrarily defined for the purposes of a certain experiment. In comparison with discrete automata (Wolfram, 1984), such assumptions imply a much broader class of solutions. The conducted research is aimed at finding an answer to the question whether there are any general regularities or patterns describing the dynamics of the assumed model.

Method

Let there exist a matrix M, whose components are called cells. Let there exist a function G, which transforms matrix M into matrix M' in a way that:

$$c_i' = G(\delta_i^j c_j, r)$$

, where c stands for the cell's value, while i and j mean coordinates, and r means a radius within which the neighbourhood of a certain cell has an influence on its value's transformation. Furthermore, to eliminate the problem of boundary values, let us assume that the boundaries of the matrix are glued together so that topology of the described space is identical with a torus.

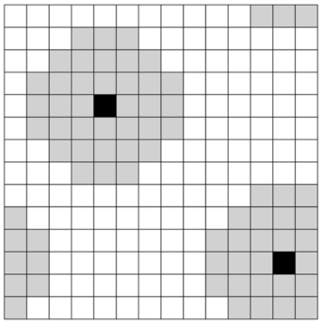

Figure 1: The value of cells established on the basis of the values of their neighbours located within the radius $r = 3$. Topology of glued boundaries guarantees continuity of solutions on the edges of the matrix.

Finally, let us consider process P in which values of all the cells are changed so that a matrix obtained after the change becomes a source matrix for the next step:

$$P: M \rightarrow M' \rightarrow M'' \rightarrow M''' \rightarrow \cdots$$

Experiment

The following experiment was done. In matrix M of a size of 512 x 512 components, a random value from the range of [0,1] was attributed to each cell. In each subsequent time step, arithmetical mean $\propto$ of a value of neighbouring cells within radius r was calculated for each cell. Subsequently, on the basis of the following functions, the weight with which the neighbourhood contributes to the later change was calculated:

$$w_1: \omega = \mathrm{e}^{-\frac{(\propto - c_i)^2}{2\sigma^2}},\ w_2: \omega = -\mathrm{e}^{-\frac{(\propto - c_i)^2}{2\sigma^2}} + 1,\ w_3: \omega = 1$$

, where c_i stands for the value of the cell whose neighbourhood is analysed, while σ constitutes a parameter of the experiment defined *a priori*.

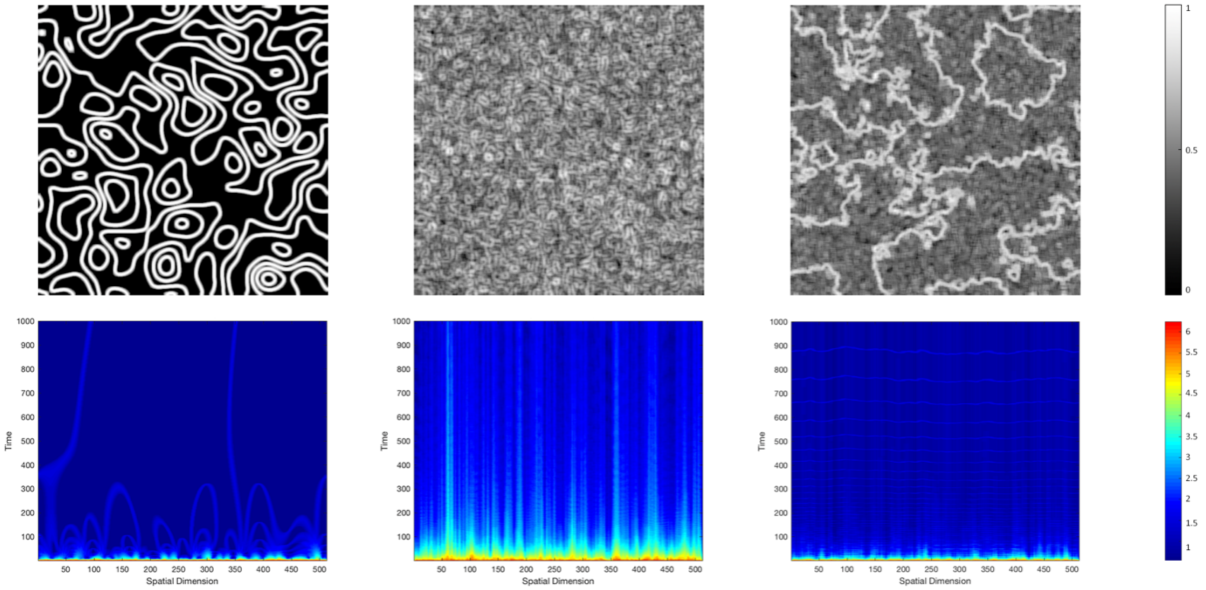

Figure 2: Normalized local entropy of the matrix (top) and local entropy in a time slice across the whole matrix and along cell no. 128 (bottom) in the experiments of the following parameters: [G_1, w_3, r=4, step of the process: 150] (left), [G_2, w_2, $\propto$=0.5, r=4, step of the process: 800) (middle), [G_2, w_2, $\propto$=0.1, r=48, step of the process: 285) (right).

The value of each cell in a subsequent time step was calculated with the use of the following functions:

$$G_1\colon c_i' = \frac{c_i + \omega\alpha}{1+\omega},\quad G_2\colon c_i' = \frac{c_i + \omega(\alpha - c_i)}{1+\omega}$$

Results

By means of the aforementioned transformations, the value of the cells of matrix M in the subsequent steps of process P was calculated. Subsequently, for each P process, proper L matrix was obtained whose every cell assumes the value equal to local entropy (MathWorks, 1994-2017) in the neighbourhood of an analogous cell of matrix M. The process of qualitative evaluation of the results distinguished three images of changes occurring within the frameworks of local entropy. Each of the obtained result could be attributed to one of these three groups. The first image is a fine, tangled pattern, almost unchanged over time; the second image is a pattern similar to broadly scattered contour lines and it varies over time in a cyclical fashion; while the third pattern is the two aforementioned patterns — unchanged and cyclical — overlapping.

Discussion

The distinguished images resemble Turing patterns occurring, among others, as the result of reaction-diffusion processes (Turing, 1952). According to the proposed hypothesis, the analysed systems might be treated as reaction-diffusion ones if low local entropy is understood as a high concentration of one component, while high local entropy as mapping of a near homogeneous mixture. Observation of a time change of Turing patterns in the dynamics of a continuous cellular automaton may show that the mechanism of imitating the neighbourhood, on which its operation relies, may imply waves of ordering patterns recurrent over time.

References

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