# IV
# SKEWNESS IN RETURN RATE DISTRIBUTION MODELLING

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## 4.1. Introduction

The appropriate modelling of return rate distribution for specific financial instruments is one of the most commonly analysed concepts related to capital market. The results of the modelling are often used as an element in a number of tools and methods used to prepare analyses, diagnoses and forecasts related to specific phenomena on financial markets. In the light of previous research, the *a priori* adoption of certain assumptions as to the form of the return rate distribution density function including, in particular, the assumption that it takes the form of a standard Gaussian function (as in the case of Markovitz model or CAMP model) seems a very risky thing to do. Additionally, it is indicated that typical properties of a return rate include the leptokurtic and fat-tailed distribution of return rates, autocorrelation, clustering, leverage effect, volatility long memory process and skewness (Pionek, 2005).

The main objective of the study is to assess the extent to which inclusion of the possibility of skewness in the density function affects the quality of return rate distribution modelling. Three basic distribution types shall be used to this end, i.e. normal distribution, Student's t-distribution and GED distribution. Additionally, the significance of skewness for modelling normal and logarithmic return rate shall be determined, as well as differences in return rate distribution skewness on both mature and developing capital markets. For this purpose, return rates on the S&P500 and WIG indices shall be subjected to modelling.

## 4.2. Methodological grounds

Skewness of each symmetrical distribution with known density function *f(x)* can be obtained through the appropriate conversion of the distribution to the left or to the right from the modal value. Then, each half of the distribution obtained constitutes a fragment of the base symmetrical distribution with different properties (Pionek, 2005). In general, the density function *h(x)* for skew distribution can be presented as follows (Arellano-Valle, R. B. et al., 2005):

$$h(x|\alpha) = \frac{2}{a(\alpha)+b(\alpha)}\left[f\left(\frac{x}{a(\alpha)}\right)I_{(x\geq 0)} + f\left(\frac{x}{b(\alpha)}\right)I_{(x<0)}\right] \qquad \text{(II.1)}$$

where $\alpha$ is the skewness (asymmetry) parameter, and $a(\alpha)$ and $b(b)$ are the functions which introduce asymmetry to the distribution.

Assuming that the density function for a normal symmetrical distribution (*norm*) is described by the formula:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right) \tag{II.2}$$

normal skew distribution (*snorm*) can be presented as (Azzalini, 1985):

$$f(x) = \frac{2}{\sigma}\varphi\left(\frac{x-\mu}{\sigma}\right)\Phi(\alpha\cdot\frac{x-\mu}{\sigma}) \tag{II.3}$$

or

$$f(x) = \frac{1}{\sigma\cdot\pi}\cdot e^{-0.5\cdot\left(\frac{x-\mu}{\sigma}\right)^2}\cdot\int_{-\infty}^{\alpha\cdot\left(\frac{x-\mu}{\sigma}\right)} e^{-\frac{t^2}{2}}dt \tag{II.4}$$

where μ –location parameter, σ-scale parameter, α-skewness parameter, ϕ(x) – density function for standardised normal distribution, Φ(x) – cumulative distribution function (CDF) for normal distribution.

The standard, symmetrical form (*t-stud*) of Student's t-distribution is described with the following density function:

$$f(x) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\sqrt{n\pi}}\left(1+\frac{x^2}{n}\right)^{-\frac{n+1}{2}} \tag{II.5}$$

while the skew Student's t-distribution (*st-stud*) can be presented as (Jones, 2003):

$$f(x) = \frac{1}{2^{a+b-1}\cdot B(a,b)\cdot(a+b)^{0.5}}\cdot\left(1+\frac{x}{(a+b+x^2)^{0.5}}\right)^{a+0.5}\cdot\left(1-\frac{x}{(a+b+x^2)^{0.5}}\right)^{b+0.5} \tag{II.5}$$

where Γ(·) – gamma function, B(·,·) – beta function, a,b>0. If a=b, then the function (II.5) is converted into standard Student's t-function with "2*a" degrees of freedom; if a<b, function negatively skewed function is obtained; while if a>b, positively skewed function is obtained.

The GED density function, also known as the GGD (*Generalized Gaussian Distribution*), is described with the use of the following formula (Purczyński, 2003):

$$f(x) = \frac{\lambda\cdot s}{2\Gamma\left(\frac{1}{s}\right)} exp(-\lambda^s|x-\mu|^s) \tag{II.6}$$

where:

$$\lambda = \frac{1}{\sigma}\left[\frac{\Gamma\left(\frac{3}{s}\right)}{\Gamma\left(\frac{1}{s}\right)}\right]^{0.5} \tag{II.7}$$

The skew GED (sGED) function, in turn, is described as follows (Theodossiou, 2000):

$$f(x) = \frac{C}{\sigma} \cdot \exp\left(-\frac{1}{[1-\mathrm{sign}(x-\mu+\delta\sigma)\lambda]^s \Theta^s \sigma^s} \cdot |x-\mu+\delta\sigma|^s\right) \quad (II.8)$$

where:

$$C = \frac{k}{2\Theta}\Gamma\left(\frac{1}{k}\right)^{-1},$$

$$\Theta = \Gamma\left(\frac{1}{k}\right)^{0.5}\Gamma\left(\frac{3}{k}\right)^{-0.5} S(\lambda)^{-1},$$

$$\delta = 2\lambda A S(\lambda)^{-1},$$

$$S(\lambda) = \sqrt{1+3\lambda^2-4A^2\lambda^2},$$

$$A = \Gamma\left(\frac{2}{k}\right)\Gamma\left(\frac{1}{k}\right)^{-0.5}\Gamma\left(\frac{3}{k}\right)^{-0.5},$$

s- shape parameter, λ – scale parameter.

The functions above shall be used in the attempt to model daily (D), weekly (W) and monthly (M) distributions of return rates, where the normal rate of return shall be calculated as:

$$R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} \quad (II.9)$$

and logarithmic rate of return $R^*_t$ as:

$$R^*_t = \ln\frac{P_t + D_t}{P_{t-1}} \quad (II.10)$$

where:
$P^t$ – price of the security in *t* period;
$P^{t-1}$ – price of the security in *t-l* period;
$D^t$ – value of dividend paid in *t* period.

This means that the following shall be subjected to modelling:

- normal daily rate of return in the period of 252 index listings ($R^t$_D_252);
- logarithmic daily rate of return in the period of 252 index listings ($R^{*t}$_D_252);
- normal daily rate of return in the period of 126 index listings ($R^t$_D_126);
- logarithmic daily rate of return in the period of 126 index listings ($R^{*t}$_D_126);

- normal weekly rate of return in the period of 52 index listings ($R^t$_W_52);
- logarithmic weekly rate of return in the period of 52 index listings ($R^{*t}$_W_52);
- normal monthly rate of return in the period of 36 index listings ($R^t$_M_36);
- logarithmic monthly rate of return in the period of 36 index listings ($R^{*t}$_M_36);
- normal monthly rate of return in the period of 60 index listings ($R^t$_M_60);
- logarithmic monthly rate of return in the period of 60 index listings ($R^{*t}$_M_60).

The above-mentioned rates of return will be modelled starting from the first index listing, in which such modelling is possible (e.g. for the daily rate of return in the period of 252 index listings, the first model was obtained for listing no. 254 for the index, while the second model, for listing no. 255, was based on return rates from listings 3 to 254 etc.), until the final listing in 2015. This means that, in the case of the S&P500 index, for each of the distributions analysed, 11353 models were obtained for daily return rates in the period of 252 listings and 11479 for 126 listings, 490 models were obtained for monthly return rates in the period of 60 listings and 514 for 36 listings and 2185 models were obtained for weekly return rates. When it comes to the WIG index, in turn, the number of models obtained equalled 5062 and 5188, 194 and 218 and 943 respectively.

The main research hypothesis adopted in the study (H1) is as follows: adjustment for skewness in modelling return rate distribution significantly increases the probability that the output distribution will consistent with the assumed one. Additionally, two auxiliary hypotheses have been proposed: (H2) probability that the distribution obtained shall be consistent with the assumed one does not differ significantly, in terms of statistics, for developed and developing markets; (H3) probability that the distribution obtained shall be consistent with the assumed one is highly dependent on the assumed time of estimations and type of the return rate modelled.

## 4.3. Research results

In order to verify the hypothesis concerning the impact of adjustment for skewness on the quality of modelling the return rate distribution, parameters have been calculated for all the previously mentioned distribution variants. In the next step, a chi-squared test was used to verify the number of the obtained *n* distributions for which there are no grounds to reject the hypothesis that they are consistent with the theoretical distribution analysed (*p-value*=0.05). Then, based on this very same test, *p-value* was determined for two indicators of the structure, at which it can be assumed that there is a statistically significant difference in probability that the distribution obtained will be consistent with the assumed one, between the symmetrical and the skew variant of the given distribution. The results are presented in Table 3.1..

**Table 3.1. Statistics of the modelled return rate distribution**

| Distribution | n | Number of good-fit distributions | | p-value for two indicators of the structure used in the text |
|---|---|---|---|---|
| | | symmetrical | skew | |
| T_stud_wig_Rt_D_252 | 5062 | 4450 | 4449 | 0.97569 |
| T_stud_wig_R*t_D_252 | 5062 | 4454 | 4459 | 0.87830 |
| T_stud_wig_Rt_D_126 | 5188 | 4573 | 4622 | 0.12986 |
| T_stud_wig_R*t_D_126 | 5188 | 4572 | 4604 | 0.32595 |
| T_stud_wig_Rt_M_60 | 194 | 168 | 168 | 1.00000 |
| T_stud_wig_R*t_M_60 | 194 | 190 | 192 | 0.41057 |
| T_stud_wig_Rt_W_52 | 943 | 806 | 822 | 0.28365 |
| T_stud_wig_R*t_W_52 | 943 | 803 | 817 | 0.35435 |
| T_stud_wig_Rt_M_36 | 218 | 190 | 189 | 0.88703 |
| T_stud_wig_R*t_M_36 | 218 | 198 | 201 | 0.60616 |
| T_stud_SP500_Rt_D_252 | 11353 | 10092 | 9980 | 0.02028 |
| T_stud_SP500_R*t_D_252 | 11353 | 10094 | 9952 | 0.00339 |
| T_stud_SP500_Rt_D_126 | 11479 | 10099 | 10196 | 0.04559 |
| T_stud_SP500_R*t_D_126 | 11479 | 10102 | 10199 | 0.04537 |
| T_stud_SP500_Rt_M_60 | 490 | 404 | 406 | 0.86601 |
| T_stud_SP500_R*t_M_60 | 490 | 406 | 393 | 0.28455 |
| T_stud_SP500_Rt_W_52 | 2185 | 1779 | 1893 | 0.00000 |
| T_stud_SP500_R*t_W_52 | 2185 | 1787 | 1894 | 0.00001 |
| T_stud_SP500_Rt_M_36 | 514 | 414 | 406 | 0.53452 |
| T_stud_SP500_R*t_M_36 | 514 | 411 | 400 | 0.40051 |
| norm_wig_Rt_D_252 | 5062 | 2915 | 3097 | 0.00023 |
| norm_wig_R*t_D_252 | 5062 | 2906 | 3068 | 0.00106 |
| norm_wig_Rt_D_126 | 5188 | 3836 | 4017 | 0.00003 |
| norm_wig_R*t_D_126 | 5188 | 3826 | 3988 | 0.00022 |

| | | | | |
|---|---|---|---|---|
| norm_wig_Rt_M_60 | 194 | 119 | 119 | 1.00000 |
| norm_wig_R*t_M_60 | 194 | 188 | 191 | 0.17687 |
| norm_wig_Rt_W_52 | 943 | 701 | 771 | 0.00009 |
| norm_wig_R*t_W_52 | 943 | 692 | 759 | 0.00024 |
| norm_wig_Rt_M_36 | 218 | 134 | 152 | 0.06712 |
| norm_wig_R*t_M_36 | 218 | 186 | 195 | 0.17961 |
| norm_SP500_Rt_D_252 | 11353 | 6354 | 6407 | 0.47832 |
| norm_SP500_R*t_D_252 | 11353 | 6353 | 6388 | 0.63970 |
| norm_SP500_Rt_D_126 | 11479 | 7890 | 8317 | 0.00000 |
| norm_SP500_R*t_D_126 | 11479 | 7883 | 8290 | 0.00000 |
| norm_SP500_Rt_M_60 | 490 | 355 | 341 | 0.32424 |
| norm_SP500_R*t_M_60 | 490 | 347 | 340 | 0.62357 |
| norm_SP500_Rt_W_52 | 2185 | 1666 | 1813 | 0.00000 |
| norm_SP500_R*t_W_52 | 2185 | 1664 | 1810 | 0.00000 |
| norm_SP500_Rt_M_36 | 514 | 368 | 376 | 0.57486 |
| norm_SP500_R*t_M_36 | 514 | 351 | 366 | 0.30737 |
| GED_wig_Rt_D_252 | 5062 | 4705 | 693 | 0.00000 |
| GED_wig_R*t_D_252 | 5062 | 4696 | 575 | 0.00000 |
| GED_wig_Rt_D_126 | 5188 | 4881 | 2724 | 0.00000 |
| GED_wig_R*t_D_126 | 5188 | 4879 | 2645 | 0.00000 |
| GED_wig_Rt_M_60 | 194 | 169 | 82 | 0.00000 |
| GED_wig_R*t_M_60 | 194 | 192 | 146 | 0.00000 |
| GED_wig_Rt_W_52 | 943 | 873 | 742 | 0.00000 |
| GED_wig_R*t_W_52 | 943 | 875 | 724 | 0.00000 |
| GED_wig_Rt_M_36 | 218 | 193 | 149 | 0.00000 |
| GED_wig_R*t_M_36 | 218 | 203 | 183 | 0.00265 |
| GED_SP500_Rt_D_252 | 11353 | 10731 | 2034 | 0.00000 |
| GED_SP500_R*t_D_252 | 11353 | 10753 | 1868 | 0.00000 |
| GED_SP500_Rt_D_126 | 11479 | 10712 | 5365 | 0.00000 |
| GED_SP500_R*t_D_126 | 11479 | 10710 | 5179 | 0.00000 |
| GED_SP500_Rt_M_60 | 490 | 457 | 247 | 0.00000 |
| GED_SP500_R*t_M_60 | 490 | 453 | 231 | 0.00000 |
| GED_SP500_Rt_W_52 | 2185 | 1947 | 1746 | 0.00000 |
| GED_SP500_R*t_W_52 | 2185 | 1945 | 1719 | 0.00000 |
| GED_SP500_Rt_M_36 | 514 | 462 | 373 | 0.00000 |
| GED_SP500_R*t_M_36 | 514 | 460 | 352 | 0.00000 |

Source: own work.

Two major conclusions can be drawn based on the results presented in Table 3.1.: (1) adjustment for skewness in modelling return rate distribution significantly increases the probability that the output distribution will be consistent with the assumed one; (2) skewness significance depends on the type of the adopted distribution modelling: for Student's t-distribution, adjustment for skewness had significant impact on the results obtained only in the case of daily and weekly return rates (both normal and logarithmic) for S&P500. When it comes to the remaining distributions analysed, such impact was observed in all cases.

Validity of the conclusions above is also confirmed by results of the Kolmogorov-Smirnov test (K_S), used to verify the hypothesis that empirical distributions of the *p-values* obtained for the chi-squared goodness-of-fit test are identical when modelling both symmetrical and skew density functions of the distributions analysed (Table 3.2.).

**Table 3.2. Results of the Kolmogorov-Smirnov (K_S) test of equality of empirical CDFs of the obtained p-values (chi-squared goodness-of-fit test), for modelling symmetrical and skew variants of the distribution**

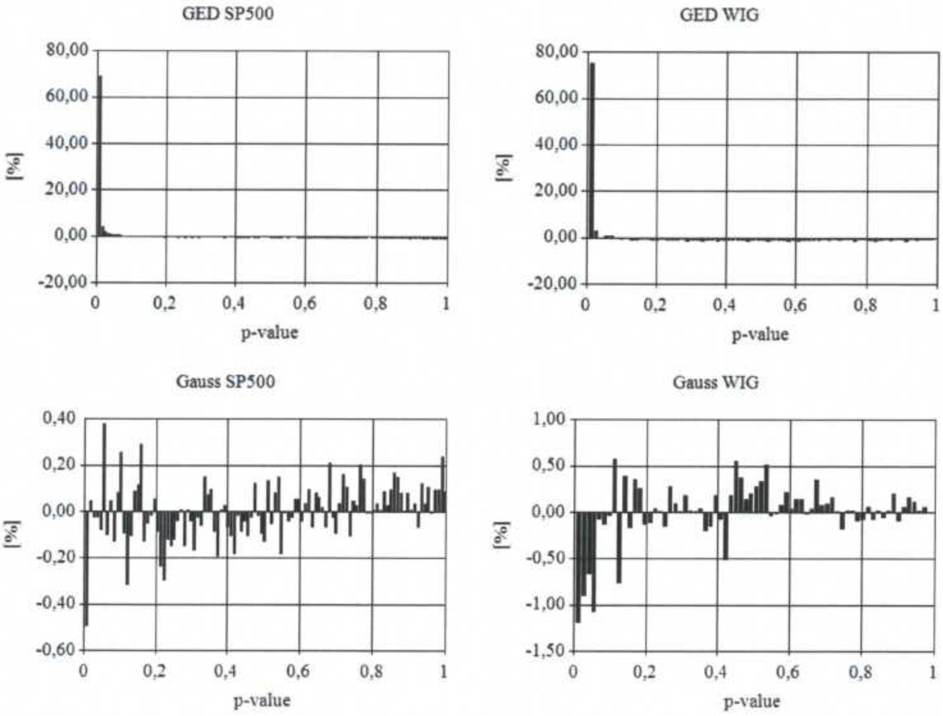
| Distribution | K_S | p-value | Distribution | K_S | p-value |
|---|---|---|---|---|---|
| T_stud_wig_Rt_D_252 | 1.79870 | 0.00310 | norm_SP500_Rt_D_252 | 1.86705 | 0.00188 |
| T_stud_wig_R*t_D_252 | 1.79870 | 0.00310 | norm_SP500_R*t_D_252 | 1.74711 | 0.00446 |
| T_stud_wig_Rt_D_126 | 2.26754 | 0.00007 | norm_SP500_Rt_D_126 | 3.09977 | 0.00000 |
| T_stud_wig_R*t_D_126 | 2.44423 | 0.00001 | norm_SP500_R*t_D_126 | 3.10839 | 0.00000 |
| T_stud_wig_Rt_M_60 | 0.75955 | 0.61111 | norm_SP500_Rt_M_60 | 0.54249 | 0.93016 |
| T_stud_wig_R*t_M_60 | 1.21529 | 0.10426 | norm_SP500_R*t_M_60 | 0.86327 | 0.44539 |
| T_stud_wig_Rt_W_52 | 3.29105 | 0.00000 | norm_SP500_Rt_W_52 | 3.86752 | 0.00000 |
| T_stud_wig_R*t_W_52 | 2.99187 | 0.00000 | norm_SP500_R*t_W_52 | 3.81956 | 0.00000 |
| T_stud_wig_Rt_WM_36 | 1.76793 | 0.00386 | norm_SP500_Rt_M_36 | 1.88337 | 0.00166 |
| T_stud_wig_R*t_WM_36 | 1.38567 | 0.04298 | norm_SP500_R*t_M _36 | 1.68624 | 0.00678 |
| T_stud_SP500_Rt_D_252 | 1.58602 | 0.01307 | GED_wig_Rt_D_252 | 39.93917 | 0.00000 |
| T_stud_SP500_R*t_D_252 | 1.30067 | 0.06786 | GED_wig_R*t_D_252 | 41.07206 | 0.00000 |
| T_stud_SP500_Rt_D_126 | 3.27337 | 0.00000 | GED_wig_Rt_D_126 | 23.47052 | 0.00000 |
| T_stud_SP500_R*t_D_126 | 3.06219 | 0.00000 | GED_wig_R*t_D_126 | 24.85461 | 0.00000 |
| T_stud_SP500_Rt_M_60 | 0.73396 | 0.65421 | GED_wig_Rt_M_60 | 5.16497 | 0.00000 |
| T_stud_SP500_R*t_M_60 | 1.08498 | 0.18974 | GED_wig_R*t_M_60 | 5.51943 | 0.00000 |
| T_stud_SP500_Rt_W_52 | 3.75069 | 0.00000 | GED_wig_Rt_W_52 | 3.84340 | 0.00000 |
| T_stud_SP500_R*t_W_52 | 3.50871 | 0.00000 | GED_wig_R*t_W_52 | 4.67192 | 0.00000 |
| T_stud_SP500_Rt_M_36 | 1.77605 | 0.00364 | GED_wig_Rt_M_36 | 4.39593 | 0.00000 |

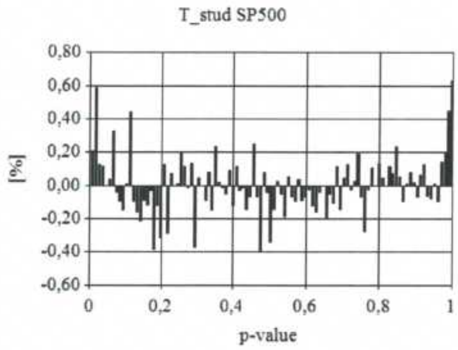
| | | | | | |
|---|---|---|---|---|---|
| T_stud_SP500_R*t_M_36 | 1.71374 | 0.00562 | GED_wig_R*t_M_36 | 2.15018 | 0.00019 |
| norm_wig_Rt_D_252 | 2.21465 | 0.00011 | GED_SP500_Rt_D_252 | 58.29782 | 0.00000 |
| norm_wig_R*t_D_252 | 2.10935 | 0.00027 | GED_SP500_R*t_D_252 | 59.27332 | 0.00000 |
| norm_wig_Rt_D_126 | 3.26674 | 0.00000 | GED_SP500_Rt_D_126 | 39.24750 | 0.00000 |
| norm_wig_R*t_D_126 | 3.46187 | 0.00000 | GED_SP500_R*t_D_126 | 40.23083 | 0.00000 |
| norm_wig_Rt_M_60 | 0.55374 | 0.91901 | GED_SP500_Rt_M_60 | 6.79710 | 0.00000 |
| norm_wig_R*t_M_60 | 0.89184 | 0.40409 | GED_SP500_R*t_M_60 | 7.37151 | 0.00000 |
| norm_wig_Rt_W_52 | 3.40080 | 0.00000 | GED_SP500_Rt_W_52 | 3.99267 | 0.00000 |
| norm_wig_R*t_W_52 | 3.08112 | 0.00000 | GED_SP500_R*t_W_52 | 4.79423 | 0.00000 |
| norm_wig_Rt_WM_36 | 1.43170 | 0.03316 | GED_SP500_Rt_WM_36 | 3.27168 | 0.00000 |
| norm_wig_R*t_WM_36 | 1.31659 | 0.06243 | GED_SP500_R*t_WM_36 | 3.58327 | 0.00000 |

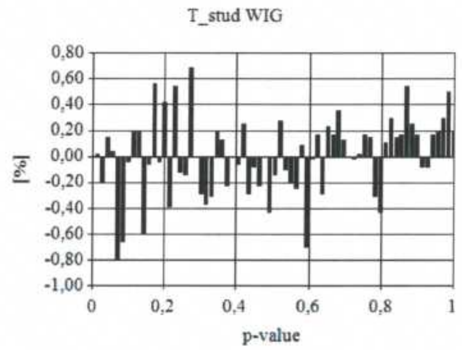
Source: own work.

Figure 3.1., in turn, shows differences in the incidence of distributions consistent with the assumed distribution, for daily return rate modelling, given the estimation time encompassing 252 index listings (positive value denotes predominance of skew distributions, negative value denotes predominance of symmetrical distributions).

**Figure 3.1. Differences in the incidence of distributions consistent with the assumed ones when modelling symmetrical and skew variants of the distribution (estimation time: 252)**






Source: own work.

In order to verify the additional hypothesis (H2), i.e., that probability that the distribution obtained shall be consistent with the assumed one does not differ significantly, in terms of statistics, for developed and developing markets, the results of rate modelling for S&P500 and WIG were compared. The results obtained, however, are inconclusive. For daily return rate modelling, significant differences in the incidence of distributions consistent with the assumed ones were reported for each GED and normal distribution, while they were absent in the case of Student's t-distribution. When it comes to weekly return rate modelling, significant differences were observed for skew GEDs and Student's t-distributions. Details concerning the results of the test are presented in Table 3.3..

**Table 3.3. P-values for two structure indicators, for tests measuring equality of the incidence of distributions consistent with the assumed distribution for return rates on WIG and SP500 indices**

| Distribution | p-value for two structure indicators in the test | Distribution | p-value for two structure indicators in the test |
|---|---|---|---|
| T_stud_Rt_D_252 | 0.97667 | snorm_Rt_D_252 | 0.05344 |
| T_stud_R*t_D_252 | 0.43913 | snorm_R*t_D_252 | 0.08374 |
| T_stud_Rt_D_126 | 0.61112 | snorm_Rt_D_126 | 0.00000 |
| T_stud_R*t_D_126 | 0.84074 | snorm_R*t_D_126 | 0.00000 |
| T_stud_Rt_M_60 | 0.22997 | snorm_Rt_M_60 | 0.00452 |
| T_stud_R*t_M_60 | 0.00000 | snorm_R*t_M_60 | 0.00000 |
| T_stud_Rt_W_52 | 0.68645 | snorm_Rt_W_52 | 0.25327 |
| T_stud_R*t_W_52 | 0.97378 | snorm_R*t_W_52 | 0.09883 |
| T_stud_Rt_M_36 | 0.01447 | snorm_Rt_M_36 | 0.00695 |
| T_stud_R*t_M_36 | 0.00000 | snorm_R*t_M_36 | 0.00000 |

| | | | |
|---|---|---|---|
| sT_stud_Rt_D_252 | 0.06737 | GED_Rt_D_252 | 0.00000 |
| sT_stud_R*t_D_252 | 0.08592 | GED_R*t_D_252 | 0.00000 |
| sT_stud_Rt_D_126 | 0.75750 | GED_Rt_D_126 | 0.00000 |
| sT_stud_R*t_D_126 | 0.82179 | GED_R*t_D_126 | 0.00000 |
| sT_stud_Rt_M_60 | 0.18624 | GED_Rt_M_60 | 0.05478 |
| sT_stud_R*t_M_60 | 0.00000 | GED_R*t_M_60 | 0.00000 |
| sT_stud_Rt_W_52 | 0.00602 | GED_Rt_W_52 | 0.43636 |
| sT_stud_R*t_W_52 | 0.02195 | GED_R*t_W_52 | 0.23921 |
| sT_stud_Rt_M_36 | 0.03129 | GED_Rt_M_36 | 0.24842 |
| sT_stud_R*t_M_36 | 0.00032 | GED_R*t_M_36 | 0.00002 |
| norm_Rt_D_252 | 0.00000 | sGED_Rt_D_252 | 0.00008 |
| norm_R*t_D_252 | 0.00000 | sGED_R*t_D_252 | 0 .00000 |
| norm_Rt_D_126 | 0.00000 | sGED_Rt_D_126 | 0.06280 |
| norm_R*t_D_126 | 0.00000 | sGED_R*t_D_126 | 0.07091 |
| norm_Rt_M_60 | 0.05637 | sGED_Rt_M_60 | 0.00923 |
| norm_R*t_M_60 | 0.00000 | sGED_R*t_M_60 | 0.00092 |
| norm_Rt_W_52 | 0.44885 | sGED_Rt_W_52 | 0.00280 |
| norm_R*t_W_52 | 0.13720 | sGED_R*t_W_52 | 0.00115 |
| norm_Rt_M_36 | 0.39706 | sGED_Rt_M_36 | 0.58582 |
| norm_R*t_M_36 | 0.00000 | sGED_R*t_M_36 | 0.12479 |

Source: own work.

Nevertheless, there is an argument for falsification of the additional hypothesis (H2), i.e., the results of the Kolmogorov-Smirnov (K_S) test of equality of the *p-value* distribution. Only in 8 out of the 60 distributions analysed did the result (p-value=0.05) indicate that *p-value* distributions for the chi-squared goodness-of-fit test, for S&P500 and WIG index, do not differ in a statistically significant way (Table 3.4.).

**Table 3.4. Results of the Kolmogorov-Smirnov (K_S) test of equality of empirical CDFs of the obtained p-values for the chi-squared goodness-of-fit test, for SP500 and WIG index return rate modelling**

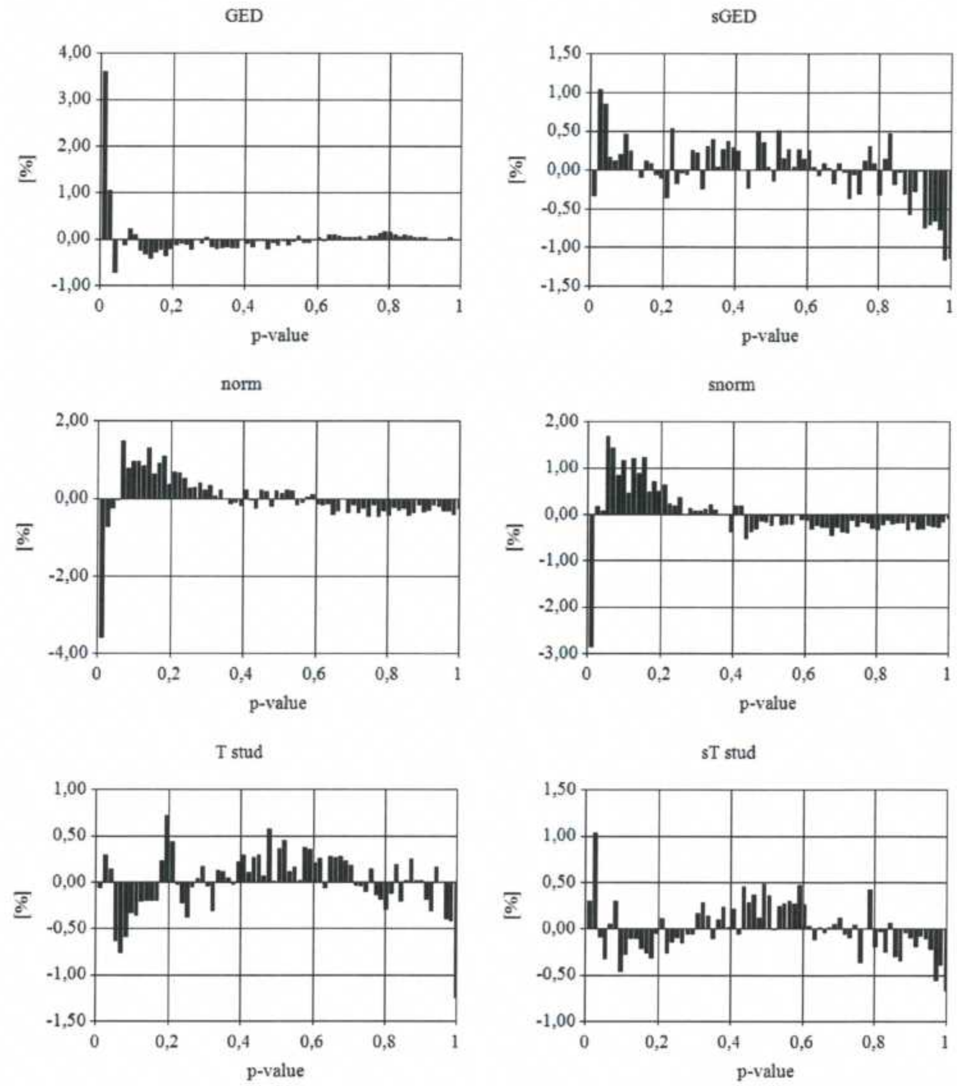
| Distribution | K_S | p-value | Distribution | K_S | p-value |
|---|---|---|---|---|---|
| T_stud_Rt_D_252 | 1.87982 | 0.00170 | snorm_Rt_D_252 | 6.05133 | 0.00000 |
| T_stud_R*t_D_252 | 1.78762 | 0.00335 | snorm_R*t_D_252 | 6.30134 | 0.00000 |
| T_stud_Rt_D_126 | 2.19446 | 0.00013 | snorm_Rt_D_126 | 3.51597 | 0.00000 |
| T_stud_R*t_D_126 | 2.06942 | 0.00038 | snorm_R*t_D_126 | 3.51505 | 0.00000 |
| T_stud_Rt_M_60 | 1.60839 | 0.01133 | snorm_Rt_M_60 | 2.33034 | 0.00004 |
| T_stud_R*t_M_60 | 5.00161 | 0.00000 | snorm_R*t_M_60 | 4.90796 | 0.00000 |

| | | | | | |
|---|---|---|---|---|---|
| T_stud_Rt_W_52 | 1.12291 | 0.16054 | snorm_Rt_W_52 | 2.15601 | 0.00018 |
| T_stud_R*t_W_52 | 1.16569 | 0.13202 | snorm_R*t_W_52 | 2.22648 | 0.00010 |
| T_stud_Rt_M_36 | 2.59381 | 0.00000 | snorm_Rt_M_36 | 1.57607 | 0.01391 |
| T_stud_R*t_M_36 | 3.94457 | 0.00000 | snorm_R*t_M_36 | 3.07158 | 0.00000 |
| sT_stud_Rt_D_252 | 2.17393 | 0.00016 | GED_Rt_D_252 | 5.85846 | 0.00000 |
| sT_stud_R*t_D_252 | 2.29730 | 0.00005 | GED_R*t_D_252 | 5.29639 | 0.00000 |
| sT_stud_Rt_D_126 | 2.75000 | 0.00000 | GED_Rt_D_126 | 4.17679 | 0.00000 |
| sT_stud_R*t_D_126 | 2.73156 | 0.00000 | GED_R*t_D_126 | 3.74895 | 0.00000 |
| sT_stud_Rt_M_60 | 2.41832 | 0.00002 | GED_Rt_M_60 | 2.01335 | 0.00060 |
| sT_stud_R*t_M_60 | 5.17387 | 0.00000 | GED_R*t_M_60 | 3.69417 | 0.00000 |
| sT_stud_Rt_W_52 | 1.07727 | 0.19616 | GED_Rt_W_52 | 1.14976 | 0.14212 |
| sT_stud_R*t_W_52 | 1.13971 | 0.14880 | GED_R*t_W_52 | 1.46205 | 0.02782 |
| sT_stud_Rt_M_36 | 2.02669 | 0.00054 | GED_Rt_M_36 | 2.14385 | 0.00020 |
| sT_stud_R*t_M_36 | 2.66602 | 0.00000 | GED_R*t_M_36 | 1.95481 | 0.00096 |
| norm_Rt_D_252 | 5.18463 | 0.00000 | sGED_Rt_D_252 | 4.09063 | 0.00000 |
| norm_R*t_D_252 | 5.38824 | 0.00000 | sGED_R*t_D_252 | 4.59241 | 0.00000 |
| norm_Rt_D_126 | 3.10792 | 0.00000 | sGED_Rt_D_126 | 2.86745 | 0.00000 |
| norm_R*t_D_126 | 3.05065 | 0.00000 | sGED_R*t_D_126 | 3.00894 | 0.00000 |
| norm_Rt_M_60 | 2.24781 | 0.00008 | sGED_Rt_M_60 | 1.69044 | 0.00659 |
| norm_R*t_M_60 | 5.60017 | 0.00000 | sGED_R*t_M_60 | 5.46260 | 0.00000 |
| norm_Rt_W_52 | 2.04769 | 0.00046 | sGED_Rt_W_52 | 1.26132 | 0.08301 |
| norm_R*t_W_52 | 2.10164 | 0.00029 | sGED_R*t_W_52 | 1.22193 | 0.10094 |
| norm_Rt_M_36 | 1.05661 | 0.21418 | sGED_Rt_M_36 | 2.53776 | 0.00001 |
| norm_R*t_M_36 | 3.35163 | 0.00000 | sGED_R*t_M_36 | 2.70756 | 0.00000 |

Source: own work.

Differences in empirical CDFs of the p-value for the chi-squared goodness-of-fit test are mainly due to the fact that lower values are far more frequently obtained for the WiG index than for S&P500, which is shown by Figure 3.2 (positive bar value means that for the specific *p-value*, the probability to obtain a distribution fitting well the assumed distribution is higher for WIG than for S&P500).

**Figure 3.2. Differences in the incidence of distributions consistent with the assumed ones when modelling return rates for SP500 and WIG indices (estimation time: 252 daily return rates)**




Source: own work.

## 4.4. Conclusion

Based on the deductions made in the article, the following conclusions may be drawn:

- admitting the potential occurrence of skewness when modelling return rates (e.g. on market indices) essentially increases the probability that the distribution obtained shall be consistent with the assumed one,
- the significance of skewness is highly dependent on the type of the distribution assumed in the modelling,
- due to problems connected with modelling rates of return on market indices, mature capital markets (e.g. American) do not differ significantly from developing markets (e.g. Polish).

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# MODERN CONDITIONS OF DEVELOPMENT
## Economics and Management

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