THE APPLICATION OF COMPUTATIONAL TOOLS OF IT IN MATHEMATICAL TASKS

Dana Országhová
Slovak University of Agriculture in Nitra,
Tr. A. Hlinku 2, 949 76 Nitra, Slovak Republic,
dana.orszaghova@uniag.sk

Abstract: *Changes in university education are caused, among others, by integrating tools of information technology into the teaching and self-study of mathematics. In the paper we present some samples of applying graphical and computational tools in mathematics. We focused on the graphical interpretation of the definite integral of a function with one variable via free computational software. By method of two-sample t-test we have verified the statistical hypothesis about the significant differences between the assessment results of compulsory study subjects Mathematics IA and Mathematics IB in the bachelor economics study programs. Based on the obtained results, there is a statistically significant difference between the exam results in the evaluated study subjects.*

Keywords: mathematics tasks, application of information technology, knowledge level, exam grades, statistical hypothesis testing.

INTRODUCTION

The engineering education is performing in the era of globalization and the important question is how to educate students of engineer study programs to be successful in the changing world and to cope with the different aspects of information explosion. In the competitors' fight it is possible to be winner with permanent innovations and high quality production.

Now that the issue of integrating technological tools into the teaching and learning of mathematics has become urgent, one can wonder what the existing theoretical perspectives have to offer. Can they be applied to this new context? Technological devices have become smaller and handheld devices such as graphing and symbolic calculators are widespread. Communication has become a more integrated part of technology use: software can be distributed using the Internet, and students can work, collaborate, and communicate with peers and teachers in digital learning environments (Drijvers et al., 2009).

The current time is considered as information age in which the conditions, content and methods of education change significantly. Information technology (IT) as an important phenomenon of the information society has become part of university education and its tools are used in a variety of ways. Teachers of mathematics use IT tools to promote active participation of students in education, to create electronic learning resources, to create assignments, to verify the correctness of the solution, or for graphical interpretation of tasks. Students can apply IT tools in solving exercises, during seminar work and projects, or in an individual mathematics study during the exam preparation.

A critical part of students' development and persistence as engineers is their acquisition of a professional identity. Prior research indicates that science, technology, engineering, and math (STEM) students tend to over calibrate their level of professional identity (Villanueva and Nadelson, 2017).

The part of the creation of an information society is education with new qualities and modern methods, including the use of multimedia and virtual elements in education (Országhová et al., 2010). One component of the work of university teachers of mathematics is the implementation of modern methods in the study system and the creation of electronic courses in mathematics (Gregáňová, 2009). E-learning courses as a practical application of new educational approaches and methods provide opportunities for the application of the specific requirements of individual subjects (Országhová, 2009).

When searching for free software, we found interesting information for students on a website (For Students - Calculators on the Web) with direct links to calculators that can be used by students to check the correctness of the result after the calculation, e.g.:

1. Function calculator (calculates function derivation, Taylor polynomial, iterations and displays function graph).

2. Matrix calculator (for a given matrix, it calculates the inverse matrix, its determinant and simple expressions).

3. Matrix multiplier (calculates matrix product).

4. Linear solver (solves a system of linear algebraic equations).

5. Solucia (solves differential equations).

6. Alcula (counts linear and quadratic regression).

An important goal of education is to provide students with the latest knowledge via modern methods, including e-learning through information technology. E-learning in education can take many forms and thus perfectly adapt to the needs of individual students, groups and organizations. As it is known, learning can be referred to as the blended if about 50 per cent of the learning interaction is mediated by ICT (Sekret and Hrubý, 2013). Blended learning is connecting the traditional way of education with the multimedia assisted learning. The "mobile" learning, especially mobile learning involving mobile telephony, is seen as becoming a new sector of education and training.

The result of mathematics study is the mastering of mathematical methods and procedures that students will use to solve economic questions from practice. This area includes various analytical tasks such as market conditions that are important for enterprises for the involvement in international business (Mura et al., 2012), the analysis of consumer opinions and preferences when buying food products (Vietoris et al., 2016), the creation of appropriate production and consumption models; the accounting and fiscal issues related to agricultural land in the case of the Slovak Republic (Krajčírová et al., 2016), and so on.

1. MATHEMATICS TOPICS IN ECONOMICS STUDY PROGRAMS

The content of compulsory subjects in mathematics at the Faculty of Economics and Management of the Slovak University of Agriculture (SUA) in Nitra is underlying the main aim – to teach the students basic knowledge of higher mathematics and to point out the possibilities of the application of the mathematical apparatus in professional subjects and in practice. Mathematics also develops logical thinking, which is necessary when solving the different situations and problems in the professional or private life.

In Table 1 we summed up the thematic units of mathematics subjects taught at the Faculty of Economics and Management. Study subjects Mathematics IA and Mathematics IB are obligatory and students have to take the exam. The evaluation system includes: a partial test during the semester, a seminar project and an exam test. Students are expected to have prepared printed literature for individual study and electronic educational courses created in LMS Moodle.

Table 1.

Main mathematics topics taught at the Faculty of Economics and Management of the Slovak University of Agriculture in Nitra

Mathematics IA, 1st year, winter term	**Mathematics IB, 1st year, summer term**
Function with one real variable	Indefinite integral
Limit of a function with one real variable	Definite integral and applications
Derivative of a function with one real variable	Linear algebra: vectors, matrices, systems of linear equations
Function with two real variables	Theory of probability

Source: Own work

2. MATHEMATICAL TASKS SOLVED WITH IT TOOLS

Applied tasks are important motivating factors of the study of mathematics for economists and managers. The applications of mathematical methods can be demonstrated to students by means of tasks with different levels of difficulty. Students can lose logical relationships in time-consuming applied tasks and this could result in the distraction from studying. Table 2 lists IT tools and software applied in mathematical topics and tasks.

Table 2.

IT tools and software used in teaching and individual study of Mathematics IA and Mathematics IB

MS Excel	Calculations with expressions, values of functions, probability, functions graphs
GraphSight v.2.0.1.	Graphs of functions with one variable, limits and asymptotes
WolframAlpha (free)	Derivative of a function, indefinite and definite integrals, graph of a function with two real variables. Linear algebra: vectors, matrices, systems of linear equations
GeoGebra	Functions, local extremes and graphs

Source: Own work

2.1 Definite integral and graphic interpretation

Indefinite and definite integrals have many applications. We focused on the area of a plane figure which has application in economics as consumer and producer surplus. The plane figure is usually bounded by two functions. The formulation of the task is: evaluate the area of a plane body bounded by functions $f: y = x^2 - 4x$, $g: y = 2x - 5$ and sketch the graphs (Figure 1).

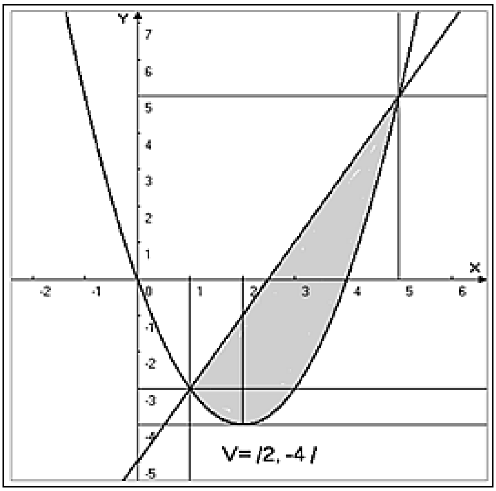

Figure 1. Graphs of functions created in the software GraphSight v.2.0.1.
Source: GraphSight v.2.0.1., own work

In the Graphics menu, program GraphSight v.2.0.1. offers the following options:

- Graph in Cartesian coordinate with expression $y = f(x)$ or $x = f(y)$,
- Graph for function given parametrically,
- Graph for function in polar coordinates,
- Table chart.

Students consider the creation of images and function charts as a difficult task, so they prefer graphic software for graphs. This approach develops their ability to combine knowledge and apply graphics programs in mathematics as well. A maths teacher can apply this program very effectively when creating new assignments for exercises, testing student knowledge, and creating test assignments. The graphs of functions are the means for the presentation of events and processes from different scientific areas and from different areas of practical life. Solving of applied tasks with graphical interpretation of a function enables students to acquire knowledge and skills that they can utilize in other specialized economic subjects.

2.2 Indefinite and definite integrals calculated in free software WolframAlpha

The following outputs were created in free software WolframAlpha. The formulation of the task is: find indefinite integral and evaluate the definite integral.

Case I

Indefinite integral:

$$\int (x^2 - 4x)\, dx = \frac{x^3}{3} - 2x^2 + \text{constant}$$

Definite integral:

$$\int_0^4 (x^2 - 4x)\, dx = -\frac{32}{3} \approx -10.667$$

Visual representation of the integral:

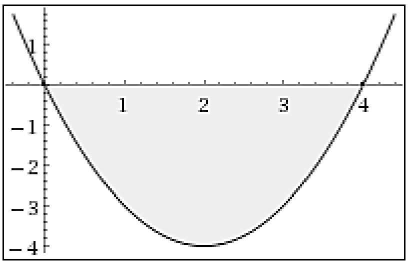

Figure 2. Graph of the function $y = x^2 - 4x$.

Source: WolframAlpha, own work

To compare two different cases we solved the task formulated as a case II. The bounded area is under axis x (case I) and above axis x (case II). Therefore the definite integral is minus or plus according to the situation.

Case II

Indefinite integral:

$$\int (-x^2 + 4x)\, dx = 2x^2 - \frac{x^3}{3} + \text{constant}$$

Definite integral:

$$\int_0^4 (-x^2 + 4x)\, dx = \frac{32}{3} \approx 10.667$$

Visual representation of the integral:

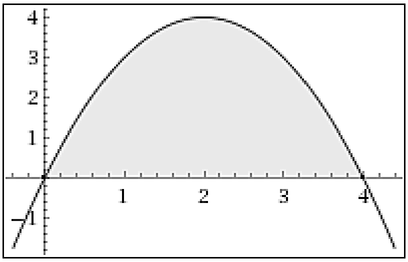

Figure 3. Graph of the function $y = -x^2 + 4x$.

Source: WolframAlpha, own work

Part-time students have fewer hours of contact study than full-time students. Mutual communication with teachers confirms that these students use the created study materials in electronic form. Mathematical educational tools for individual study are accessible through courses created by the Department of Mathematics (the Slovak University of Agriculture in Nitra) in the LMS Moodle environment and include examples with the solving procedures and tasks for self-study with solutions. Students can join the university's web sites at the time that suits them best for study.

3. ANALYSIS OF STUDY OUTPUTS IN MATHEMATICS

3.1 Analysis of tasks solving

Presented examples of solved tasks confirm that their solution is conditioned by the combining knowledge of mathematics: calculus and geometry. The data for analysis were obtained from the compulsory subjects Mathematics IA (winter term) and Mathematics IB (summer term) in the 1st study year of the bachelor's degree at the Faculty of Economics and Management.

We selected mathematical tasks from the partial and final tests with topics: integral calculus and linear algebra. The themes in the assignments were as follows:

Task 1 (T1): inverse matrix,

Task 2 (T2): system of linear equations,

Task 3 (T3): the area of a plane figure (use of a definite integral),

Task 4 (T4): graphs of functions,

Task 5 (T5): indefinite integral.

In Figure 4 the tasks are presented with the percentage rank of earned points. Students gained better results in solving the tasks from linear algebra (T1, T2). They achieved the smallest number of points in solving the task with graphs of functions (T4). The decrease in the number of mathematics hours at secondary schools resulted in the reduction in the scope of analytical geometry.

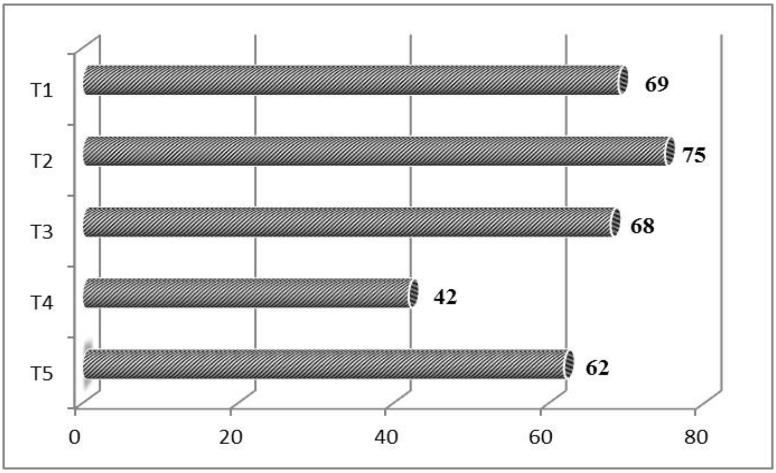

Figure 4. Results of tasks analysis expressed in a percentage.

Source: Own work

3.2 Mathematics exam grades

In this part we present the results of the analysis of exam grades of mentioned subjects Mathematics IA and Mathematics IB taught in academic year 2015/2016. In Figure 5 it is displayed in the graphic form of evaluation with the standard scale from A(1) to FX(4). We see that the final evaluation of knowledge by the grade E(3) is the most common in both subjects. The comparison shows that in the summer semester (Math IB), students have worse evaluation grades. This fact is caused by leaving from the university study after the first semester; the reason for leaving is also insufficient knowledge from the secondary school. Moreover, students do not have a great ambition to get a better grade in the summer semester.

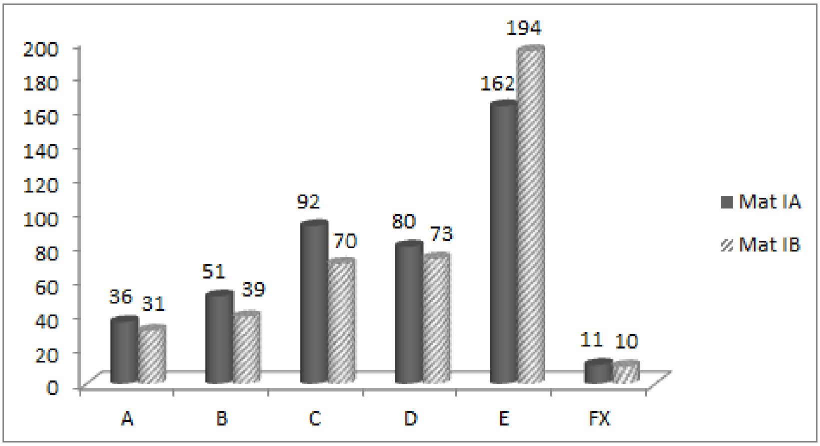

Figure 5. Comparison of final grades in Math IA and Math IB.

Source: Own work

Using the two-sample t-test in MS Excel, we tested the null hypothesis about the differences in the knowledge level of students represented by final grades in the study subjects Math IA and Math IB.

Table 3.

Results of statistical testing of exam grades of subjects Mathematics IA and IB

t-Test: Two-Sample Assuming Equal Variances		
	Mat IA	*Mat IB*
Mean	2.334	2.442
Variance	0.444	0.427
Observations	421	407
Pooled Variance	0.433	
Hypothesized Mean Difference	0	
df	826	
t Stat	-2.374	
P(T<=t) one-tail	0.009	
t Critical one-tail	1.647	
P(T<=t) two-tail	0.018	
t Critical two-tail	1.963	

Source: Own work

In Table 3 there are presented the results obtained. Because the calculated value (two-tail) $P = 0.018 < 0.05$, the null hypothesis about the equal knowledge level of students in the given subjects at the significance level $\alpha = 0.05$ cannot be accepted. Therefore, we can state that there is statistically significant difference between the exam results achieved by the students in Mathematics IA and Mathematics IB.

4. DISCUSSION

The implementation of IT tools into the educational process at universities also means changes in the system of education. Students have to work more actively, individually, search for the necessary information and to discover relevant relationships more easily with IT. The education with usage of new technologies is more flexible and widely available at any location and with access anytime (Hornyák, Gregáňová, 2016).

New opportunities are open by using IT in mathematics:

- Display of function graphs,
- Rationalizing complex and time-consuming calculations,
- Graphical interpretation of solved tasks,
- Development of spatial imagination of students,
- Using the appropriate application or program (e. g. MS Excel, Mathematica, GraphSight v.2.0.1., GeoGebra, free WolframAlpha, etc.).

As Turek (2006) states, the transition from an industrial society to an information society (learning society) is characterized by the fact that in the industrial society the capital is the main strategic source of development, while in the information society knowledge and information become the source for the progress. This results in new learning objectives: to develop the ability of students (people) to create and use information, to orientate and to apply them, or in other words, it is necessary to develop an interest in learning, to be able to learn effectively throughout the whole life and to adapt flexibly to the changing circumstances of our life.

It is obvious that teachers are the important component in this process, so the change in the preparation of future teachers is also needed.

All educational institutes should implement effective tools for knowledge management in order to sustain in competitive world. Barbara Friehs (2003) mentioned following assignments for the effective usage of knowledge management:

- Mobilize the hidden implicit/tacit knowledge,
- Integrate knowledge from organization and make it accessible to all,

- Identify the missing knowledge, create new knowledge,
- Make knowledge more accessible and usable,
- Create knowledge sharing culture to experiment and learn,
- Evaluate and reflect learning processes,
- Codify new knowledge.

CONCLUSION

- One of the many advantages of implementation of IT tools into the process of mathematics education is better explanation and visualization of the mathematical concept. A key factor in the functioning of the information society is education, whose aim is to ensure that people are able to find and understand the information, then apply it correctly.
- In the paper we focused on topics with graphical interpretation of functions and integrals via software products. This method should help students studying at the university (from different types of secondary schools) to realize computations and display graphs.
- The evaluation of knowledge level of students is the important part of pedagogical research. By two-sample t-test we analysed the exam grades in two obligatory study subjects: Mathematics IA (winter term) and Mathematics IB (summer term) taught in the first study year at the Faculty of Economics and Management. The statistical results obtained proved that there is a statistically significant difference between the exam results in these evaluated study subjects. In this sample the study outcomes are worse in Mathematics IB.
- By solving tasks with IT support we improve and develop:
  - Students knowledge level to apply theoretical principles of mathematics,
  - Activity, autonomy and creativity of students,
  - Permanent and systematic knowledge,
  - Interdisciplinary relationships,
  - Quality of learning outcomes.

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Author's declaration

I, Dana Országhová (author of this contribution) honestly declare that I send own original work for publishing, not printed before in other sources in the same form.