DETAILED HORIZONTAL GEODETIC CONTROL NETWORKS TAKING INTO ACCOUNT THE ACCURACY OF THE REFERENCE POINTS

Edward Preweda[1]

[1] AGH University of Science and Technology, **Poland**

ABSTRACT

Regardless of the nature and type of geodetic control networks, their task is to create a base for determining the position of various objects on the surface of the Earth. In the whole globe there is a need to conduct detailed geodetic measurements and to develop their results in the binding spatial reference system. In Poland, three classes of accuracy of terrain details are designated. Of all the field details, the highest accuracy of position determination is required for border points. This results from the need to guarantee, with adequate accuracy, ownership rights within certain limits and to protect property interests of real estate parties. In practice, border points, in the case of classical measurement methods, are determined on the basis of detailed or measurement networks, which are aligned with the assumption that the connection points are flawless. At the same time, the current legal provisions relate to the accuracy of determining the location of the boundary point to the geodetic control network of the 1st class. Therefore, it is necessary to use a different approach to the problem of leveling horizontal geodetic control networks and determining the accuracy of terrain details, also taking into account the accuracy of the points of establishment. The paper presents the results of research and conclusions resulting from this approach to compaction of detailed networks.

Keywords: horizontal geodetic control networks, determining the accuracy

Wprowadzenie

According to the Regulation [11], a detailed horizontal geodetic network consists of:

- points of the previous horizontal 2 class network, whose average position error in relation to the reference points after alignment $m_p \leq 0{,}05$ m;
- points of the previous horizontal 3 class network, whose average position error in relation to the reference points after alignment $m_p \leq 0{,}10$ m;
- newly assumed horizontal network points, whose average position error in relation to the reference points after alignment $m_p \leq 0{,}07$ m.

Points of the detailed horizontal geodetic control network are assumed in the networks [5], using observations of static GNSS satellite measurements, measurements performed as part of the ASG-EUPOS system and classical measurements using polygonisation and indents [7]. Detailed GPS networks, both in urban and rural areas, measured by the static method, allow to determine the positions of points with accuracy of even a few millimeters. The development of measurement techniques using GPS satellite technology has also made it possible to increase the efficiency of measurements. The RTK (Real Time Kinematic) method allows real-time relative measurements (based on a reference station), in any defined flat coordinate system, with an accuracy of 1 to 3 cm. This work focuses on classical

measurements using the polygonization and indentation method. Measurements are made in many surveying works and constitute a very important part of processes related to both individual plots [2] and entire complexes - eg the external boundary of land covered by merger and land exchange [4], [9].

According to [11], horizontal geodetic network connection should be established to all points of the basic network located in the area of the study. In justified cases, when combining existing geodetic networks, it is permissible to refer to the points of the detailed network, provided that the number of such references does not exceed 30% of the total number of references. In order to integrate the new network with the existing matrix in the field, the control points of the same class with known coordinates should be included in the measurement.

Observations are leveled in the strictest way, using the least-squares method, assuming the correctness of the reference points. In the case of establishing a detailed network to the points of the 1st or 2nd class warp, the adoption of the correctness of the reference points does not generate greater distortion of the coordinates and assessment of the accuracy of the detailed network. However, when referring to the same class of warp points, the adoption of the error of the reference points makes the impact of the inaccuracy of the reference points significantly deforms the observations, which are determined relatively more accurately than the previously determined points of reference. This is particularly important in the case of modernization of a fragment of a detailed network or its density for current needs, when the entire network from a given poviat, for example, only a part of it, is not compensated again. In this case, it is practically impossible to obtain the required accuracy of the position of the points at the required accuracy $mp \leq 0{,}07$ m.

It should be emphasized that the adoption of zero errors of the reference points means that although the alignment is carried out using the least squares method, the idea of this method is distorted in this case and the obtained results may differ far from those that should actually be obtained by the least squares method. Thus, it can be stated that the provision specified in the Regulation [10] "Observations aligns in the strictest way, using the least squares method, assuming the correctness of the points of reference" only defines the general idea of levelling the detailed network, because its strict implementation is simply not possible. The least squares exact alignment would be only if all the points included in the network, also the points of reference, constituted a free network and so should be treated during the network equalization.

Practical aspect

According to the Regulation [11], the networks of lower classes are aligned with the necessity of referring to the points of the higher-class networks, or even networks of the same class, with the co-ordinates of the reference points to be unmistakable. Establishing a network is necessary due to the need to maintain the same scale and orientation in the whole state network, which was given to 1-class networks and to give all points of the network coordinates in a uniform state coordinate system. The assumption of the error of the points of establishment is accepted for practical reasons, primarily to keep the coordinates of the state network points cataloged in the databases. However, it should be emphasized that by assuming the coordinates of the reference points as error-free, we should assume that errors in the determination of these

coordinates will not cause significant deformation of the observation material, and changes in the assessment of their accuracy caused by the introduction of conditions for changing the size of the designated network elements will not have practical meaning . However, if the measurement accuracy of the network is established in the same order as the accuracy of the measurement of the network to which points it binds, then these changes will be significant. In order to limit the impact of errors in the relative position of the reference points on the size of the observation corrections, the coordinates and their accuracy characteristics to be determined, it should be aligned with the rejection of the assumption of the accuracy of the reference points. Introduction when aligning any conditions of connection will always change both in the coordinate values and in the evaluation of the accuracy of the designated network elements. The selection of specific reference points will cause such changes [1], [3].
In the case of compaction of the 3rd class network, in order to minimize the impact of reference point errors on the determined coordinates, the inaccuracy of the reference points can be taken into account, assuming coordinates of reference for pseudo-observations (previously processed observations), but of a known covariance matrix. The lack of such data significantly limits the possibility of using full information on the accuracy of coordinates, which nowadays can be stored without any technical obstacles in the geodetic databases.

In terms of computational capabilities, determining the full covariance matrix does not cause any technical or technical problems. However, until full information on coordinate accuracy is mandatory and available in the databases on state geodetic control networks, only the approximation of the inaccuracy of the reference points can be estimated. It is necessary to take the same values of standard deviations along both axes of the coordinate system. Such assumptions are at odds with the idea of a strict equalization account. In practice, however, in the absence of having complete data, it is advantageous to use even incomplete and poor information to improve the quality of coordinate information and their accuracy, rather than consistently recognize the points of reference as unmistakably determined.

Horizontal network compensation, taking into account the erroneous coordinates of the reference points

Detailed 3 class networks, considered in the poviat scale, can range from several to several dozen thousand points, depending on the area and extent of the poviat area investment. In the case of modernization of the detailed network on such a scale, close grid alignment for a given poviat can be implemented in a simultaneous manner. Due to the observations linking the given network with the neighboring districts, the scope of the network is growing, at least to the nearest, outside the poviat boundary, class 1 and 2 points. In order to densify the horizontal class 3 detailed network for current needs or for its partial modernization, classic geodetic measurements are often made using polygonization and indents. The problem of equalizing observations with the method of least squares is reduced in the first stage to the matrix matrix system of form equations $\mathbf{Ax} = \mathbf{L} \leftarrow \mathbf{P}$ [6], [8], where

A – matrix of observation equations,
L – the vector of the difference between the measured value and the approximate value,
x – vector of unknowns,
P – weight matrix.

In order to take into account the accuracy of the coordinates of the points of reference, they can be treated as previously processed observations (pseudo-observations). Because such coordinates are both observations and uncertainties in the network equalization process, hence the relationships must follow the equalization:

$$v_{x_i} = dx_i, \quad v_{y_i} = dy_i$$

Matrix **A** and vector **L** will take the form:

	Coordinates of reference points					Coordinates of new points					L
	dx_{N_1}	dy_{N_1}	...	dx_{N_j}	dx_{N_j}	dx_{W_1}	dy_{W_1}	...	dx_{W_k}	dx_{W_k}	
Angle coefficients	...	...	...	...	...	...	...	...	...	...	$\alpha - \alpha^o$
Distance coefficients	...	...	...	...	...	...	...	...	...	...	$d - d^o$
Coefficients for coordinates of reference points	1	0	...	0	0	0	0	...	0	0	0
	0	1	...	0	0	0	0	...	0	0	0
			...					...			
	0	0	...	1	0	0	0	...	0	0	0
	0	0	...	0	1	0	0	...	0	0	0

The weight matrix results from the stochastic model, taking into account the accuracy of the observation and the accuracy of the coordinates of the reference points. The stochastic model can be saved in the form:

$$\mathbf{P} = \mathbf{Cov}(\mathrm{L},\mathrm{X})^{-1} = \left[\begin{array}{c|c} \mathbf{Cov}(\mathrm{L}) & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{Cov}(\mathrm{X}) \end{array}\right]^{-1}$$

Matrix **Cov**(L) results from the accuracy of the measured angles and length in the network. These quantities are usually dependent in a physical sense (angles and distances are usually measured by a total station, hence, for example, the centering accuracy affects both measured values), however, it is not possible to determine numerical values describing these relationships before equalization, hence the covariance between these values is accepted it's zero. The covariance matrix present in the stochastic model **Cov**(X) for the coordinates of the points of reference can be determined on the basis of a close alignment of the matrix to which we refer, or only approximated, assuming different assumptions:

$\mathbf{Cov}(\hat{\mathrm{X}}) = \hat{\sigma}^2\mathbf{W}$ - a model that takes into account the full covariance matrix for the reference points,

$\mathbf{Cov}(\hat{\mathrm{X}}) = \hat{\sigma}^2\mathbf{D}$ - assumption that the coordinates of the points of reference are uncorrelated but different accurate

$\mathbf{Cov}(\hat{\mathrm{X}}) = \hat{\sigma}^2\mathbf{E}$ - the assumption that the coordinates of the reference points are equally accurate and uncorrelated

$\mathbf{Cov}(\hat{\mathrm{X}}) = \mathbf{0}$ - assumption that the coordinates of the reference points are error-free.

If the coordinates of the reference points are assumed to be error-free, the matrix of coefficients is reduced to the form:

	Coordinates of new points					L
	dx_{W_1}	dy_{W_1}	...	dx_{W_k}	dx_{W_k}	
Angle coefficients	...	...	...	...	...	$\alpha - \alpha^o$
Distance coefficients	...	...	...	...	...	$d - d^o$

The weight matrix then takes the form: **P**=**Cov**(L)$^{-1}$. The solution of the system (1) by the least squares method is known by vector:

$$\hat{\mathbf{x}} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{L}$$

In the area of measurement accuracy, Regulation [10], [11] requires the use of geodetic instruments that provide an average measurement error of direction less than 20^{cc}. The average length measurement error should not be greater than 0,01 m. It also allows adaptation of observations from old measurements, the average error of which does not exceed twice the error value of the average measurement provided for the modernized specific network.

Assessment of the accuracy of the density of the detailed matrix

Considerations on the possible achievable accuracy of determining the location of compacted points for the current needs of the detailed network were carried out on the example of a fragment of the network located in the Krakow poviat (Figure 1).

Fig. 1. Area of research

There are several class 2 warp points in the study area and newly established class 3 points with an error mp<= +- 7 cm. The concept of densification of the 2nd class warp with the accuracy of the points of reference in order to verify the possible accuracy of the location of the points is based on the assumption that they are made for immediate needs:

- 2-nd class points have been combined in such a way as to separate segments (Figure 3), inside which various methods of network density have been designed with points of the detailed network;
- each segment is considered separately, taking into account the erroneousness of the class 2 reference points (m_p=±1 cm);
- finally, the network was completely aligned for the whole of the study area

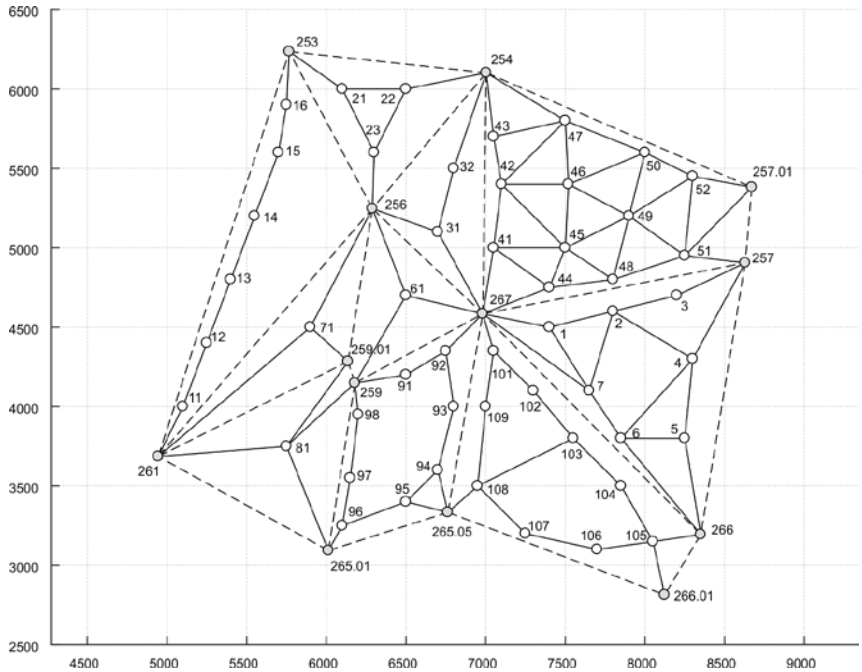

Fig. 2. Sketch of the second class network and the design of its density

Table 1 presents the accuracy characteristics for selected points of the detailed network, in the case of separate compensation within the segments. It should be noted that in the case of total network equalization, better accuracy characteristics are obtained. Fig. 3 and 4 show ellipses of probability constant probability errors and position errors of the detailed class 3 network points.

Table 1. The accuracy characteristics for determining the coordinates of selected points

Nr pkt	m_X [cm]	m_Y [cm]	m_p [cm]	A [cm]	B[cm]	Az[g]
1	0,9	0,8	1,2	0,9	0,8	9,4
14	1,5	1,9	2,4	1,9	1,4	119,2
21	0,8	0,7	1,1	0,8	0,7	155,3
32	0,9	0,8	1,2	0,9	0,8	15,5
41	1,0	1,3	1,6	1,3	1,0	116,9
51	1,3	0,8	1,5	1,3	0,8	190,8
61	0,9	1,0	1,4	1,0	0,9	81,2
71	1,0	1,1	1,4	1,2	0,8	137,0
81	0,7	0,7	1,0	0,7	0,7	155,1
91	0,7	0,9	1,1	0,9	0,6	80,1
101	0,9	0,7	1,1	0,9	0,7	179,5
102	1,0	0,9	1,4	1,0	0,9	164,4

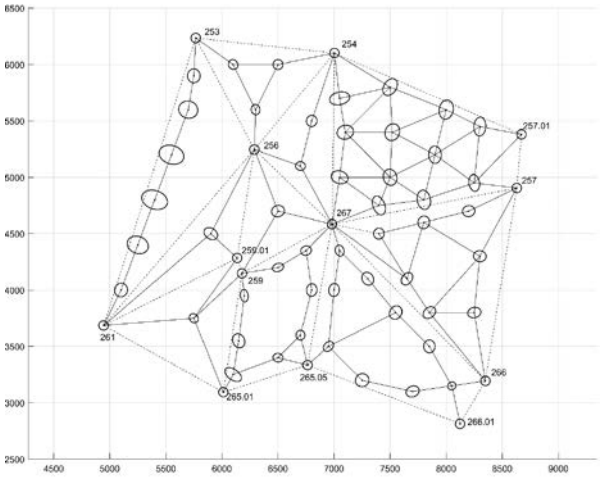

Fig. 3. Ellipses of errors in the 3rd class network points

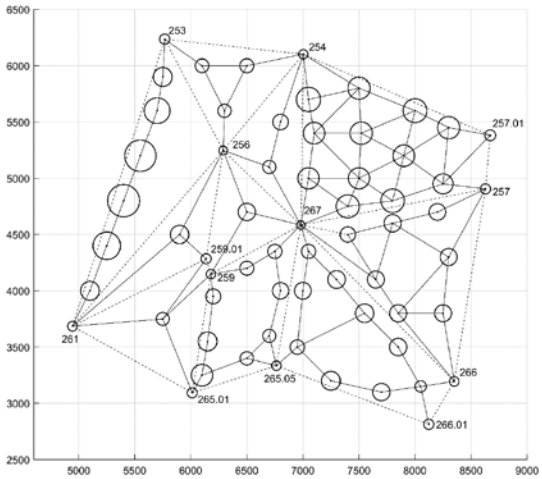

Fig. 4. Errors in the location of the 3rd class specific network points

Based on the calculations carried out, it is stated that the maximum position error of the point does not exceed 2.5 cm. With the exception of three points, all others are characterized by the accuracy of the point below 2.0 cm. Thus, all designed points meet, with excess, the criteria of the detailed class 3rd . If strict information on the accuracy of coordinates of points is not recorded in the geodetic databases, then for further references it will be necessary to assume the maximum permissible error of the position of the warp point of that class, i.e. ±7 cm.

Conclusions

Taking into account the technological progress and high financial outlays incurred in order to obtain the highest accuracy of measurement, impoverishment of information on the accuracy of the coordinates designated points at the stage of entering this information into databases is inappropriate. Full information expressed in the covariance matrix is necessary in order to carry out a strict assessment of the accuracy of the location of points determined based on the geodetic network. The problem of geodetic control alignment is known, however, the significance of the covariance matrix for the coordinates of the reference points should be emphasized and the possibility of using it to account for the erroneousness of the reference points. An important problem related to the inaccuracy of the reference points is the fact that the coordinate values of the reference points also change as a result of the adjustment, on the basis of which the coordinates of other points constituting the basis for land and building registry were often designated earlier. However, the change of these values is so small that they do not necessarily have to be updated in databases after each density of the matrix for small areas. Considering that the location of border points and their accuracy should be related to the basic 1 warp, it is necessary to take into account the inaccuracy of the warp.

Funding

This work was carried out within the statutory studies of the AGH University of Science and Technology, Faculty of Mining Surveying and Environmental Engineering, Department of Geomatics No. 11.11.150.006.

Bibliography

[1] Butryn K. ,Jasińska E., Preweda E. The ambiguity of the results of the strict adjustment of horizontal geodetic networks. SGEM 2016. Albena, Bulgaria, 2016. DOI: 10.5593/SGEM2016/B22/S09.079

[2] Hanus P., Jasinska E., Preweda, E. Analysis of the accuracy of determining the coordinates property borders. In Environmental Engineering. Proceedings of the International Conference on Environmental Engineering. ICEE (Vol. 9, p. 1). Vilnius Gediminas Technical University, Department of Construction Economics & Property. DOI: 10.3846/enviro.2014.209

[3] Jasińska E., Preweda E. Effect the accuracy of benchmarks to establish of the determination of geodetic network. 9th ICEE, Vilnius Gediminas Technical University Press Technika, 2014.

[4] Mika M., Siejka M., Leń P., Król Ż. 2016. The concept of using the water cadastre databases components for the construction of multi-dimensional cadastre in Poland. Survey Review, DOI: 10.1080/00396265.2016.1263180.Postek P.& Pachelski W. Design of Geodetic Network Using Controlled Computer Simulation. Baltic Geodetic Congress (BGC Geomatics), p. 138-142. DOI: 10.1109/BGC.Geomatics.2017.26

[5] Preweda E.: Rachunek wyrównawczy ⇒ modele statystyczne [Adjustment computations ⇒ statistical models]. Wyd. PROGRES, Kraków, 2013.

[6] Oleniacz G., Skrzypczak I., Swieton T. 2017. Trimble M3 1" And South Nts-362r Total Station Angle Measurement Accuracy Analysis. Civil And Environmental Engineering Reports .Volume: 24, Issue: 1. DOI: 10.1515/ceer-2017-0003

[7] Wiśniewski Z. Rachunek wyrównawczy w geodezji z przykładami. Wydawnictwo UWM Olsztyn, 2005.

[8] Wójcik-Leń J., Stręk Ż., 2017. The Share Of Land Belonging To Non-Resident Owners In A Village Intersected By A Motorway - Case Study. Geographic Information Systems Conference and Exhibition, GIS ODYSSEY 2017: Conference proceedings , 4th to 8th of September 2017, Trento – Vattaro, Italy, 2017, pp. 427-435

[9] Rozporządzenie MSWiA z 9 listopada 2011 w sprawie standardów technicznych wykonywania geodezyjnych pomiarów sytuacyjnych i wysokościowych oraz opracowywania i przekazywania wyników tych pomiarów do państwowego zasobu geodezyjnego i kartograficznego (Dz. U. z 2011 r. Nr 263, poz. 1572).

[10] Rozporządzenie MAiC z dnia 14 lutego 2012 r. w sprawie osnów geodezyjnych, grawimetrycznych i magnetycznych (Dz U. z 2012 r. Poz. 352).