Benefits of Monte Carlo simulation as the extension to the Programe Evaluation and Review Technique

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Abstract — **Given the complexity and innovation of projects, project managers are increasingly forced to take into account the aspect of risk and uncertainty in project planning. The aim of the article is to compare two project management best planning practices: the Programe Evaluation and Review Technique and Monte Carlo simulation, as well as to present benefits of employment Monte Carlo simulation as the extension to the Programe Evaluation and Review Technique.**

The research methodology included literature review of strengths and weaknesses of two methods and the analysis of PERT and Monte Carlo simulation results for estimating the budget for the construction project presented in a case study. Automated Monte Carlo simulation was modeled and performed in MS Excel with additional Monte Carlo add-on "@Risk for Excel". Key findings of the comparative study show better precision and comprehensiveness of Monte Carlo simulation in contrast to ease of use of PERT method. PERT provides better project estimates when used alongside with Monte Carlo simulation.

Keywords: project management, risk management, Monte Carlo, PERT.

I. INTRODUCTION

The variety of endeavors carried out in the form of projects led over the last half-century to the emergence of a wide range of methods and techniques of project management [1, p.38-43], [2, p.193]. Among them, special attention should be paid to the group of network project planning techniques. Their characteristic feature is representation of the project in form of a network graph consisting of nodes and edges imaging activities and events in the project [3, p.197]. These techniques were designed to assist project managers in particular in process of planning scope and time in projects in different planning situations which results from innovation and risk of particular project.

The purpose of this paper is to present and compare two techniques of project management - PERT technique and Monte Carlo simulation techniques, as well as an indication of their mutual synergies and benefits of an integrated approach. The weaknesses of PERT technique can be offset by the simultaneous use of Monte Carlo methods resulting in higher reliability of planning forecasts.

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II. PROGRAM EVALUATION AND REVIEW TECHNIQUE

The basic network techniques such as CPM (Critical Path Method) [4, p.8], MPM (Metra Potential Method) [5, p.195-203] and LOB (Line of Balance) [6, p.839] are recommended for typical, repetitive projects with comprehensively known scope, well defined, and the potential changes and risks only slightly affect the entire course of the project. In addition to the so-called "determined task structure" a second condition for the application of these techniques are precise estimates of the tasks attributes (time, cost and resources) [7, p.104]. According to the recommendations of those techniques in order to perform calculations for a project, for each activity its duration should be presented as a single value, for example 7 working days.

TABLE I. PROJECT NETWORK PLANNING TECHNIQUES

Project network structure / **Project activities atributes**	**Determined**	**Probabilistic**
Determined	• CPM • MPM • LOB	• GERT
Probabilistic	• PERT	• GERTS

Source: Trocki M. (ed.), Nowoczesne zarządzanie projektami, PWE, Warsaw 2012

Prerequisites of determined activity structure and its determined attributes entail significant limitation of employment of those methods. The reality is changeable and unpredictable [8, p.410], therefore it is very difficult and expensive to provide high quality of estimates [9, p.428]. In order to better reflect the impact of risk and uncertainty on a project and in order to increase reliability of the estimates is recommended to use PERT (Program Evaluation and Review Technique) [4, p.9].

PERT technique is well-known and widely used technique for planning. Its development was related to the

implementation of the Polaris submarine project [10, p.242] and Apollo spacecraft program in the 50's and 60's of the twentieth century [11, p.646-669], [12, p.B2]. PERT recognized as one of the best project management practice and is referred to by the main global and industry project management standards [13, p 16], [14, p.73]. PERT technique introduces a stochastic component to the project planning, assuming that the estimated values are not certain (determined), but may occur according to some probabilistic distribution. When planning a project in accordance with PERT project scheduler does not estimate activity duration pointwise (as a single value as in the case of CPM method) but using the three parameters [10, p.243]:

- a - optimistic activity duration, corresponding to the most favorable scenario of the task,
- m – the most probable value, corresponding to the most typical, dominant scenario,
- b – pessimistic value, representing an extremely unfavorable course of the task.

The above method of PERT estimating may be used for estimating the duration of the task, as well as their costs (as in the following case study) and other resource requirements [13, p.205]. On the basis of the assumptions and guidance set out in the method project manager is able to identify the expected duration of each task (weighted average activity time) (1) and its standard deviation (2). For this purpose, the PERT technique originally uses the beta distribution [13, p.17].

$$t_e = \frac{a+4m+b}{6} \quad (1)$$

$$\delta_{t_e} = \frac{b-a}{6} \quad (2)$$

Based on the expected times of individual tasks is possible to calculate the expected duration of the project (3) and its standard deviation (4).

T_e = total expected time of activities from the critical path (3)

$$\delta T_e = \sqrt{\sum \delta_{t_e}^2} \quad (4)$$

Knowing the expected project duration and standard deviation of its critical path allows the probability of completing the project by specific time to be computed using standard statistical tables. The equation below (5) is used to compute the "Z" value found in statistical tables (Z= number of standard deviations from the mean), which in turn tells the probability of finishing the project in the time specified [10, p.243].

$$Z = \frac{T_S - T_e}{\sqrt{\sum \delta_{t_e}^2}} \quad (5)$$

where: Ts – specified scheduled project duration

Te – critical path duration

Z – probability (of meeting scheduled duration) found in statistical table of normal distribution

Introducing a component of probability to project planning is undoubtedly strength of the technique. It is not, however, free of the weaker spots [15], [16, p.473], [17]. From the perspective of the purpose of this article it is worth to draw attention to the issue of narrowing field of analysis of possible options for the course of the project. According to the foundation of technique and practice of the project management, in the actual project implementation each activity can take time regarding its specific probability distribution. The PERT developers chose an approximation of the beta distribution to represent activity durations that is skewed more toward the right and is representative of work that trends to stay late one it is behind [10, p.242].

Thus, there is a significant (or in the case of continuous distributions - infinite) number of variants of the real implementation time of the activity. According to the PERT technique project scheduler simplifies the reality by choosing two variants of the extreme (a and b) and the most likely option (m). On this basis he/she calculates the weighted average activity time - expected duration of the task. That is how from the whole distribution scheduler chooses de facto only one scenario for each task execution and uses it to compute and to estimate the duration of the project. In later steps of project planning having calculated the expected project duration (T_e) and its standard deviation (δT_e), a scheduler can employ central limit theorem and use the statistics of the normal distribution to calculate the probability of completion of the project for a time specified.

This process can thus be compared to the shape of an hourglass. On the basis of the normal distribution a scheduler somehow recreates the diversity and variability of the project previously lost due to reduction of full distributions of activities duration to its average values - t_e.

This approach to the issue of probability was justified in the 50's of the twentieth century, when the PERT technique was created, and processing power of computers at that time was limited [7, p.104]. Currently, the limit is gone and the issue of uncertainty in the projects can be addressed by more sophisticated and accurate method of assessment, such as, among others Monte Carlo method. According to PMI Monte Carlo simulation can be successfully employed also if PERT assumptions do not apply (e.g. central limit theorem not applicable due to too few activities in the sequence or in case of interdependence of activity durations) [4, p.10].

III. MONTE CARLO SIMULATION

Monte Carlo technique is a technique for decision support based on multiple statistical simulations (modeling) the cumulative performance of the analyzed phenomena that entail risks. The source of technique creation was research on the development of the atomic bomb ("Manhattan District Project") carried out during the Second World War. Technique was developed by physicists, who successfully employed it to perform multiple simulations of behavior of matter particles in nuclear reactions [18], [19, p.125-130]. Name of the technique is directly related to the Monte Carlo, district of Monaco, European gambling capital, known for its casinos and beaches. This name was used for the first time in the 40's of twentieth

century by the American physicist working at Los Alamos [20, p.46].

Currently, the Monte Carlo technique is used successfully in situations where the overall progress of complex phenomena depends on the course of partial events that are non-deterministic, but are subject to stochastic volatility defined by statistical distributions [13, p.340]. These characteristics are most directly relevant to the implementation of complex and unique endeavors, i.e. projects.

In the field of the project management Monte Carlo simulation is used primarily in project risk management to estimate the risks associated with the time and the cost of the project (simulation of the costs, benefits and the level of profitability of the project) [7, p.103], [16 , p.735-742], [21, p.39]. The use of Monte Carlo simulation allows not only to find the most likely time (or budget) of the project, but also to compute their probability of occurrence of any value specified. Information gained by a scheduler are in fact similar to outputs of PERT planning process.

Monte Carlo simulation however extends the PERT technique [22, p.207], [23, p.839-860], since the estimation of project schedule (or its budget) is not based one variant of the project (in PERT – critical path computed according to the expected duration of activities), but multiple simulations of as much as 1,000, 10,000 or more runs. This allows the results to a lesser extent to be based on pure statistics and central limit theorem, and more on the random sampling and law of large numbers, which is more akin to everyday life. What is also important, the Monte Carlo simulation keeps for each task its original complexity and uncertainty in the form of their individual probability distributions. Those distributions may or may not be mutually independent.

For the use of a Monte Carlo technique is necessary to use specialized software to support this technique. This is necessary because of the need to work on distributions of variables, as well as the large number of iterations of the simulation that requires random sampling from distributions of variables. Among the available software worth mentioning are: @Risk, Risk+, RiskAMP and Monte Carlo Primavera.

A summary of the steps used in performing a Monte Carlo simulation for cost and schedule follows: [24, p.231].

1. Formulate the area and scope of the problem and the purpose of analysis (eg to estimate the necessary size of the project budget)
2. Identification of sources of data for the elements and their parameters as well as obtaining the data (eg, to determine the probability distribution of costs of the tasks on the basis of historical data from past project and / or expert judgment, and others)
3. Modeling the analyzed problem in Monte Carlo simulation software, and data input
4. Determination of the simulation parameters - the most common simulation parameter is the number of repetitions performed; depending upon the needs of the simulation it may be composed of a few, several hundreds or even several thousand repetitions; additional iterations increases the time required for their execution reaching up to quarters of an hour or longer in the case of a complex models;
5. Conducting simulations - the simulation software using (pseudo)random number generator draws of tasks parameter values making calculations according to the given model
6. Analysis of the data - after the Monte Carlo simulation software returns the results obtained with the parameters of the distribution of the resulting variable, usually these are: the number of repetitions, the mean, standard deviation, minimum, maximum, median, percentile values of the distribution. Usually, aggregated information is presented in tabular form or in the form of graphs (histograms).

Results of the Monte Carlo simulation are helpful in determining adequate levels of funding for the project or the time of its implementation as well as the necessary contingencies. On this basis, the person performing the analysis can provide answers to questions such as: what is the probability that the project will be completed at a cost of less than X? in less than Y days? how much additional reserve of time/budget should be allocated to the project in order to achieve the probability of success of Z%?

As pointed by C.I. Pritchard, this method is best suited to determine the cumulative probability of achieving the objectives of cost and/or time, but it is not very reliable in estimating the probabilities of the individual events. Hence the value of the tool lies in its ability to determine the ranges of values sought [24, p.227-235].

IV. PERT AND MONTE CARLO COMPARISON – CASE STUDY

The following example uses the PERT and Monte Carlo techniques for the analysis of the budget for the project of building a well in a random community. Project scope and activities are specified in the table below (table II). The estimates of costs for each task are also provided (in euro) according to PERT beta distribution.

TABLE II. WORK BREAKDOWN STRUCTURE OF A CASE PROJECT

	Task	a	m	b
1.	Choice of location and well pre-design	700	1000	1600
2.	Obtaining permits	400	500	700
3.	Preparation of the technical design	2700	3000	4000
4.	Purchase of materials and equipment	5500	7000	10000
5.	Site preparation and selection of the contractor	3000	4000	4800
6.	Drilling water intake	26000	3000	3400
7.	Building the foundation	6000	7000	9000
8.	Building the roof	5000	9000	15000
9.	Performing the water installation	3500	4000	4700
10.	Interior finishing	4000	4500	5300
11.	Technical acceptance	500	500	650
12.	Sanitary acceptance	500	500	650
13.	External finishing	2100	2500	3300
14.	Clearance of the construction	450	500	800
	Total	**36950**	**47000**	**63900**

(Estimates are only for illustrative purpose. No real data included).
Source: own study

The base cost of the project is €47.000. The optimistic scenario (but highly unlikely) implies the possibility of the project completion within €36.950, while the pessimistic scenario (again, highly unlikely) at a price of €63.900.

Using PERT three-point estimates expected costs and their standard deviations were calculated (table III).

TABLE III. EXPECTED COST OF ACTIVITIES

	Task	k_e – expected cost of activity	δk_e – standard deviation for expected cost of activity
1.	Choice of location and well pre-design	1050.00	150.00
2.	Obtaining permits	516.67	50.00
3.	Preparation of the technical design	3116.67	216.67
4.	Purchase of materials and equipment	7250.00	750.00
5.	Site preparation and selection of the contractor	3966.67	300.00
6.	Drilling water intake	3000.00	133.33
7.	Building the foundation	7166.67	500.00
8.	Building the roof	9333.33	1666.67
9.	Performing the water installation	4033.33	200.00
10.	Interior finishing	4550.00	216.67
11.	Technical acceptance	525.00	25.00
12.	Sanitary acceptance	525.00	25.00
13.	External finishing	2566.67	200.00
14.	Clearance of the construction	541.67	58.33

Source: own study

The total expected cost of the project is in this case, the sum of the expected costs of all tasks. This cost amounts to €48,141.68, with standard deviation - €1975.25. Probability of implementation of the project at a cost not exceeding the cost expected is due to PERT principles - 50%. According to the properties of the normal distribution, it can be estimated that there is a 68.1% probability that the budget fits within €48,141.67 +/- 1975.25., i.e. from €46,116.42 to €50,116.91.

These data were entered into a spreadsheet program with @RISK plug-in that allows the simulation of the project by the Monte Carlo technique. As a result, after 10,000 experiments average result of the project budget at the level of €48,141.68 with a standard deviation of €2216.84 was achieved. Histogram showing the simulation result is shown below (Figure 1).

Implications of Monte Carlo simulation that can be drawn by the project planner are as follows:

- average project budget for 10,000 variants of its implementation was €48,141.68, with a standard deviation of €2216.84
- in 90% of the analyzed cases, the cost of the project ranged from €44,599 to €51,878.
- its maximum value in the simulation was €55,544.79 and is much lower than the assumed worst-case scenario
- the minimum value was €41,618.63 and is much higher than the assumed optimistic scenario
- there is a 68.1% probability that the project budget will exceed 47,000 zł (!)
- to achieve 95% confidence that the project budget will be saved an additional amount of €4878.48 is requested, to a total budget of €51,878.48

V. CONCLUSIONS

The PERT technique is well known and fairly common technique for project planning. Its big advantage is the simplicity and the ability to use with little knowledge of statistical tools. Nevertheless, estimates derived using the above method are only approximation. It should be remembered that the PERT technique by calculating the expected parameters for individual tasks (budget, duration) de facto limits the options of the project to a single case based on the expected value which probability of achievement is 50%. Striving for the best quality of planning and high-precision estimates resulting directly in profit margin project managers should more frequently take advantage of a Monte Carlo simulation, which is far more precise and complex method. With little effort, with the support of the software it is possible to perform not one, but even a few thousand simulations of the project gaining precise information on the probability distribution of the project as shown in the above mentioned case.

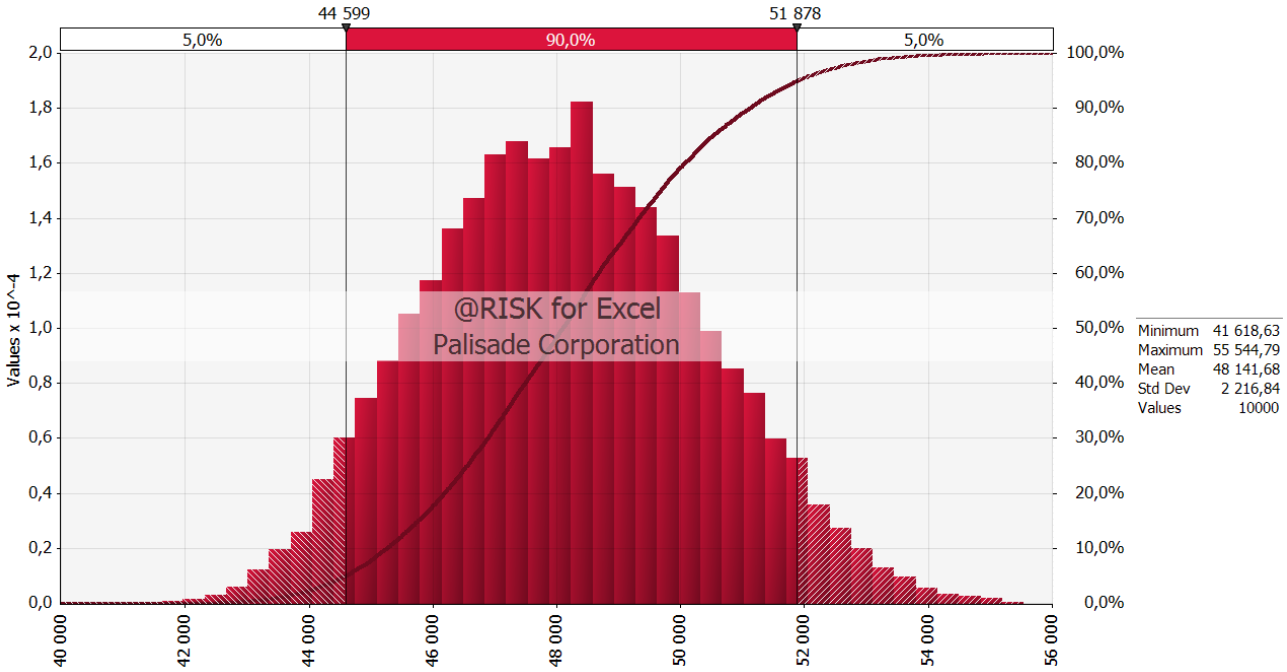

Figure 1. Monte Carlo simulation results

Source: own study using @Risk for Excel, Palisade Corporation

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